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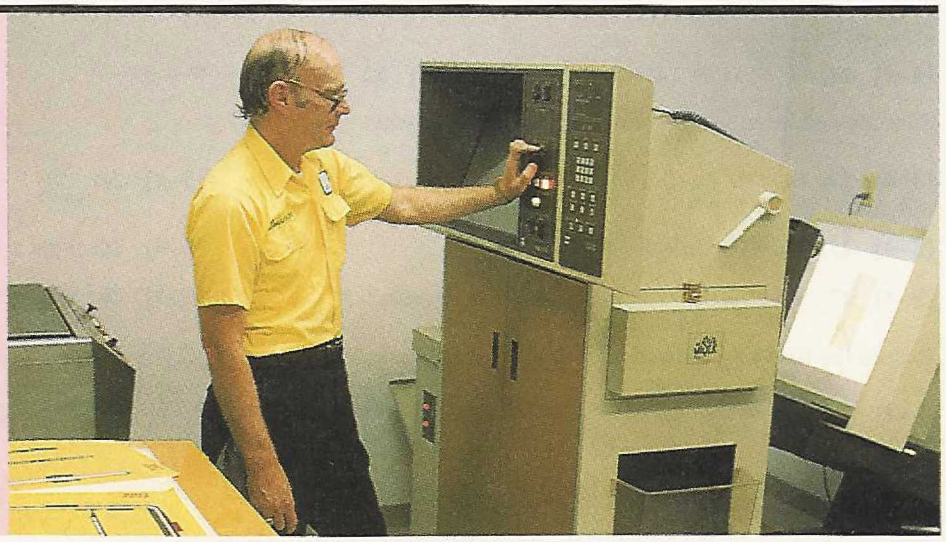
CHAPTER

6 Operations with Rational Expressions

Marc owns $\frac{5}{8}$ interest in a print shop and his uncle owns $\frac{1}{4}$ interest in the shop. In a given year, they shared earnings of \$140,000. How much did the shop earn that year?



6–1 ■ Multiplication and division of rational expressions

Multiplication of rational expressions

Recall that to multiply two real number fractions we multiply the numerators and multiply the denominators.

Multiplication property of fractions

If a, b, c, and d are real numbers, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad (b, d \neq 0)$$

Note Any possible reduction is performed *before* the multiplication takes place.

■ **Example 6–1 A**

Multiply the fractions $\frac{3}{7}$ and $\frac{14}{27}$ and simplify the product.

$$\frac{3}{7} \cdot \frac{14}{27} = \frac{3 \cdot 14}{7 \cdot 27}$$ Multiply the numerators
Multiply the denominators

$$= \frac{3 \cdot 2 \cdot 7}{7 \cdot 3 \cdot 3 \cdot 3}$$ Factor the numerator and the denominator

$$= \frac{2 \cdot (3 \cdot 7)}{3 \cdot 3 \cdot (3 \cdot 7)}$$ Group the common factors $(3 \cdot 7)$

$$= \frac{2}{3 \cdot 3}$$ Divide numerator and denominator by $(3 \cdot 7)$

$$= \frac{2}{9}$$ Multiply the remaining factors

This same procedure is followed when we multiply two rational expressions.

Multiplication property of rational expressions

Given rational expressions $\frac{P}{Q}$ and $\frac{R}{S}$, then

$$\frac{P}{Q} \cdot \frac{R}{S} = \frac{P \cdot R}{Q \cdot S} \, (Q, S \neq 0)$$

Since we want the resulting product to be stated in lowest terms, we apply the *fundamental principle of rational expressions* and divide both the numerator and the denominator by their common factors. That is, we *reduce* by dividing out the common factors.

Multiplication of rational expressions

1. State the numerators and denominators as indicated products. (Do not multiply.)
2. Factor the numerator and the denominator.
3. Divide the numerators and the denominators by the factors that are common.
4. Multiply the remaining factors in the numerator and place this product over the product of the remaining factors in the denominator.

Example 6–1 B

Perform the indicated multiplication and simplify your answer. Assume that no denominator equals zero.

1. $\frac{3x}{4} \cdot \frac{5}{2y}$

$$= \frac{3x \cdot 5}{4 \cdot 2y}$$ Multiply numerators and denominators

$$= \frac{15x}{8y}$$ Will not reduce

2. $\frac{x + 1}{x - 3} \cdot \frac{4}{x + 2}$

$$= \frac{(x + 1) \cdot 4}{(x - 3)(x + 2)}$$ Multiply numerators and denominators

$$= \frac{4x + 4}{x^2 - x - 6}$$ Will not reduce

3. $\dfrac{4}{9x} \cdot \dfrac{3x^2}{2}$

$= \dfrac{4 \cdot 3x^2}{9x \cdot 2}$ — Multiply numerators / Multiply denominators

$= \dfrac{2 \cdot 2 \cdot 3 \cdot x \cdot x}{3 \cdot 3 \cdot 2 \cdot x}$ — Factor numerator and denominator

$= \dfrac{2 \cdot x \cdot (2 \cdot 3 \cdot x)}{3 \cdot (2 \cdot 3 \cdot x)}$ — Identify common factors

$= \dfrac{2x}{3}$ — Divide numerator and denominator by $(2 \cdot 3 \cdot x)$

4. $\dfrac{x+1}{3-x} \cdot \dfrac{(x-3)^2}{x-2}$

$= \dfrac{(x+1) \cdot (x-3)^2}{(3-x) \cdot (x-2)}$ — Multiply numerators / Multiply denominators

$= \dfrac{x-3}{3-x} \cdot \dfrac{(x+1)(x-3)}{x-2}$ — Factor opposites

$= \dfrac{-1 \cdot (x+1)(x-3)}{x-2}$ — $\dfrac{x-3}{3-x} = -1$

$= \dfrac{-1(x^2-2x-3)}{x-2}$ — Multiply as indicated

$= \dfrac{-x^2+2x+3}{x-2}$

5. $\dfrac{x^2-8x+16}{x^2+3x-10} \cdot \dfrac{x^2-4}{x^2-5x+4}$

$= \dfrac{(x^2-8x+16)(x^2-4)}{(x^2+3x-10)(x^2-5x+4)}$ — Multiply numerators and denominators

$= \dfrac{(x-4)(x-4)}{(x-4)(x-2)} \dfrac{(x-2)(x+2)}{(x+5)(x-1)}$ — Factor numerator and denominator

$= \dfrac{(x-4)(x+2)}{(x+5)(x-1)}$ — Reduce by common factors $(x-4)$ and $(x-2)$

$= \dfrac{x^2-2x-8}{x^2+4x-5}$ — Multiply remaining factors

▶ ***Quick check*** Multiply $\dfrac{12}{5y} \cdot \dfrac{15y^2}{4}$ ■

Division of rational expressions

Recall that to divide two fractions $\dfrac{a}{b}$ and $\dfrac{c}{d}$, we multiply $\dfrac{a}{b}$ by the *reciprocal* of $\dfrac{c}{d}$, which is $\dfrac{d}{c}$.

Division property of fractions

If a, b, c, and d are real numbers, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c} \quad (b, c, d \neq 0)$$

Example 6–1 C

Find the indicated quotient. Reduce your answer to lowest terms.

$\frac{18}{25} \div \frac{9}{5}$

$= \frac{18}{25} \cdot \frac{5}{9}$ — Multiply by the reciprocal of $\frac{9}{5}$

$= \frac{2 \cdot 3 \cdot 3 \cdot 5}{5 \cdot 5 \cdot 3 \cdot 3}$ — Factor in numerator and denominator

$= \frac{2 \cdot (3 \cdot 3 \cdot 5)}{5 \cdot (3 \cdot 3 \cdot 5)}$ — Group the common factors $(3 \cdot 3 \cdot 5)$

$= \frac{2}{5}$ — Reduce by the common factors $(3 \cdot 3 \cdot 5)$

Division of rational expressions is done in the same way.

Division property of rational expressions

If $\frac{P}{Q}$ and $\frac{R}{S}$ are rational expressions, then

$$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} = \frac{P \cdot S}{Q \cdot R} \quad (Q, R, S \neq 0)$$

Notice that once the operation of division has been changed to multiplication, we proceed exactly as we did with the multiplication of rational expressions.

Division of rational expressions

1. Multiply the first rational expression by the reciprocal of the second.
2. Proceed as in the multiplication of rational expressions.

Example 6–1 D

Find the indicated quotients. Express the answer in reduced form.

1. $\frac{3ab}{5} \div \frac{9abc}{10}$

$= \frac{3ab}{5} \cdot \frac{10}{9abc}$ — Multiply by the reciprocal of $\frac{9abc}{10}$

$= \frac{3ab \cdot 2 \cdot 5}{5 \cdot 3 \cdot 3 \cdot abc}$ — Factor numerator and denominator

$= \frac{2 \cdot (3 \cdot 5 \cdot ab)}{3 \cdot c \cdot (3 \cdot 5 \cdot ab)}$ — Group common factors $(3 \cdot 5 \cdot ab)$

$= \frac{2}{3c}$ — Divide numerator and denominator by common factors $(3 \cdot 5 \cdot ab)$

2. $\dfrac{x^2-4}{5} \div \dfrac{x-2}{15}$

$= \dfrac{x^2-4}{5} \cdot \dfrac{15}{x-2}$ — Multiply by the reciprocal of $\dfrac{x-2}{15}$

$= \dfrac{(x-2)(x+2)\cdot 3\cdot 5}{5\cdot(x-2)}$ — Factor numerator and denominator

$= \dfrac{3\cdot(x+2)\cdot 5(x-2)}{5(x-2)}$ — Locate common factors $5(x-2)$

$= \dfrac{3(x+2)}{1} = 3x+6$ — Reduce to lowest terms by dividing numerator and denominator by $5(x-2)$

3. $\dfrac{4x+2}{x-1} \div \dfrac{2x+1}{4-4x}$

$= \dfrac{4x+2}{x-1} \cdot \dfrac{4-4x}{2x+1}$ — Multiply by the reciprocal of $\dfrac{2x+1}{4-4x}$

$= \dfrac{2(2x+1)(-4)(x-1)}{(x-1)(2x+1)}$ — Factor $4-4x=-4(x-1)$ and $4x+2=2(2x+1)$

$= \dfrac{2(-4)(2x+1)(x-1)}{(2x+1)(x-1)}$ — Reduce by $(x-1)(2x+1)$

$= 2(-4)$

$= -8$

4. $\dfrac{x^2-9}{2x+1} \div \dfrac{3-x}{2x^2+7x+3}$

$= \dfrac{x^2-9}{2x+1} \cdot \dfrac{2x^2+7x+3}{3-x}$ — Multiply by the reciprocal of $\dfrac{3-x}{2x^2+7x+3}$

$= \dfrac{(x-3)(x+3)\cdot(2x+1)(x+3)}{(2x+1)(3-x)}$ — Factor numerators

$= \dfrac{x-3}{3-x} \cdot \dfrac{(2x+1)(x+3)(x+3)}{2x+1}$ — Factor opposites

$= -1\cdot(x+3)(x+3)$ — $\dfrac{x-3}{3-x}=-1$; reduce by $(2x+1)$

$= -1(x^2+6x+9)$ — Multiply as indicated

$= -x^2-6x-9$ — Multiply by -1

▶ ***Quick check*** Divide $\dfrac{x^2-4}{4x-1} \div \dfrac{2-x}{4x^2+3x-1}$

Mastery points

Can you

- Multiply rational expressions?
- Divide rational expressions?

Exercise 6–1

Find the indicated product or quotient. Write your answer in simplest form. Assume all denominators are nonzero. See examples 6–1 A–D.

Example $\frac{12}{5y} \cdot \frac{15y^2}{4}$

Solution

$= \frac{12 \cdot 15y^2}{5y \cdot 4}$ Multiply numerator and denominator

$= \frac{3 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot y \cdot y}{5 \cdot y \cdot 2 \cdot 2}$ Factor numerator and denominator

$= \frac{3 \cdot 3 \cdot y(5 \cdot y \cdot 2 \cdot 2)}{1(5 \cdot y \cdot 2 \cdot 2)}$ Group common factors $(5 \cdot y \cdot 2 \cdot 2)$

$= \frac{9y}{1}$ Divide by common factors $(5 \cdot y \cdot 2 \cdot 2)$

$= 9y$

Example $\frac{x^2 - 4}{4x - 1} \div \frac{2 - x}{4x^2 + 3x - 1}$

Solution

$= \frac{x^2 - 4}{4x - 1} \cdot \frac{4x^2 + 3x - 1}{2 - x}$ Multiply by the reciprocal of $\frac{2 - x}{4x^2 + 3x - 1}$

$= \frac{(x + 2)(x - 2) \cdot (4x - 1)(x + 1)}{(4x - 1)(2 - x)}$ Factor numerator and denominator

$= \frac{x - 2}{2 - x} \cdot \frac{(x + 2)(x + 1)(4x - 1)}{4x - 1}$ Factor opposites

$= -1 \cdot (x + 2)(x + 1)$ $\frac{x - 2}{2 - x} = -1$; reduce by $4x - 1$

$= -x^2 - 3x - 2$ Multiply as indicated

1. $\frac{24}{35} \cdot \frac{7}{8}$
2. $\frac{3}{8} \cdot \frac{5}{9}$
3. $\frac{7}{10} \div \frac{21}{25}$
4. $\frac{56}{39} \div \frac{8}{13}$
5. $\frac{4a}{5} \cdot \frac{5}{2}$
6. $\frac{16b}{7a} \cdot \frac{5}{4}$
7. $\frac{14}{3a} \div \frac{7}{15a}$
8. $\frac{6x}{5y} \div \frac{21x}{15y}$
9. $\frac{5}{6} \cdot \frac{3x}{10y}$
10. $\frac{7a}{12b} \cdot \frac{9b}{28}$
11. $\frac{9x^2}{8} \cdot \frac{4}{6x}$
12. $\frac{36p^2}{7q} \cdot \frac{14q^2}{28p^3}$
13. $\frac{24a}{35x} \div 6a$
14. $\frac{14y}{23x} \div 7y$
15. $6a \div \frac{24a}{35x}$
16. $7y \div \frac{14y}{23}$
17. $\frac{21ab}{16c} \cdot \frac{8c^2}{3ab^2}$
18. $\frac{18x^2y^2}{5ab} \cdot \frac{25a^2b}{12xy}$
19. $\frac{5x^2}{9y^3} \div \frac{20x}{6y}$
20. $\frac{28m}{15n} \div \frac{7m^2}{3n^3}$
21. $\frac{24abc}{7xyz^2} \cdot \frac{14x^2yz}{9a^2}$
22. $\frac{80x^2yz^3}{11mn^2} \cdot \frac{33mn^2}{25xyz}$
23. $\frac{3ab}{8x^2} \div \frac{15b^3}{16x}$
24. $\frac{20mn^3}{9x^2} \div \frac{4mn}{3xy^2}$
25. $\frac{x + y}{3} \cdot \frac{12}{(x + y)^2}$
26. $\frac{5(a - b)}{8} \cdot \frac{12}{10(a - b)}$
27. $\frac{9 - p}{7} \div \frac{4(p - 9)}{21}$
28. $\frac{4x - 2}{15} \div \frac{1 - 2x}{27}$
29. $\frac{3b - 6}{4b + 8} \cdot \frac{5b + 10}{2 - b}$
30. $\frac{8y + 16}{3 - y} \cdot \frac{4y - 12}{3y + 6}$
31. $\frac{4a + 12}{a - 5} \div (a + 3)$
32. $\frac{9 - 3z}{2z + 8} \div (6 - 2z)$

33. $(x^2 - 4x + 4) \cdot \dfrac{18}{x^2 - 4}$

34. $\dfrac{21}{a^2 - 9} \cdot (a^2 + a - 12)$

35. $\dfrac{x^2 - 4}{25y} \cdot \dfrac{24y^2}{x + 2}$

36. $\dfrac{16a^2}{b^2 - 9} \cdot \dfrac{b - 3}{12a^2}$

37. $\dfrac{r^2 - 16}{r + 1} \div \dfrac{r + 4}{r^2 - 1}$

38. $\dfrac{p^2 + 2p + 1}{4p - 1} \div \dfrac{p^2 - 1}{16p^2 - 1}$

39. $\dfrac{9 - x^2}{x + y} \cdot \dfrac{4x + 4y}{x - 3}$

40. $\dfrac{b^2 - a^2}{2a + 4b} \cdot \dfrac{a + b}{a - b}$

41. $\dfrac{a^2 - 5a + 6}{a^2 - 9a + 20} \cdot \dfrac{a^2 - 5a + 4}{a^2 - 3a + 2}$

42. $\dfrac{a^2 - 5a - 14}{a^2 - 9a - 36} \cdot \dfrac{a^2 + 10a + 21}{a^2 + 4a - 77}$

43. $\dfrac{x^2 - 2x - 3}{x^2 + 3x - 4} \div \dfrac{x^2 - x - 6}{x^2 + x - 12}$

44. $\dfrac{y^2 + 3y + 2}{y^2 + 5y + 4} \div \dfrac{y^2 + 5y + 6}{y^2 + 10y + 24}$

45. $\dfrac{2x^2 - 15x + 7}{x^2 - 9x + 8} \cdot \dfrac{x^2 - 2x + 1}{x^2 - 49}$

46. $\dfrac{4x^2 - 4}{3x^2 - 13x - 10} \cdot \dfrac{x^2 - 6x + 5}{4x + 4}$

47. $\dfrac{6r^2 - r - 7}{12r^2 + 16r - 35} \div \dfrac{r^2 - r - 2}{2r^2 + r - 10}$

48. $\dfrac{4x^2 - 9}{x^2 - 9x + 18} \div \dfrac{2x^2 - 5x - 12}{x^2 - 10x + 24}$

49. $(3x^2 - 2x - 8) \div \dfrac{x^2 - 4}{x + 2}$

50. $(8a^2 - 16a) \div \dfrac{a^3 - 16a}{a - 4}$

51. $\dfrac{3x - 4}{2x + 1} \div (6x^2 - 5x - 4)$

52. $\dfrac{m^2 - 3m - 10}{m^2 - 4} \div (2m^2 - 9m - 5)$

53. $\dfrac{10}{a^3 - 27} \cdot \dfrac{a^2 + 3a - 18}{15}$

54. $\dfrac{x^3 - 8}{16} \cdot \dfrac{24}{x^2 + 2x - 8}$

55. $\dfrac{z^2 - 5z - 14}{z - 4} \div \dfrac{5z^3 + 40}{z^2 - z - 12}$

56. $\dfrac{3b^3 + 3}{b - 2} \div \dfrac{b^2 + 2b + 1}{b^2 + 6b - 16}$

57. $\dfrac{y^2 + 8y + 16}{y + 4} \cdot \dfrac{y^2 - 25}{y^2 + 9y + 20} \cdot \dfrac{y^2 + 5y}{y^2 - 5y}$

58. $\dfrac{6m^2 - 7m + 2}{6m^2 + 5m + 1} \cdot \dfrac{2m^2 + m}{4m^2 - 1} \cdot \dfrac{12m^2 - 5m - 3}{12m^2 - 17m + 6}$

Review exercises

Add or subtract the following. See section 1–1.

1. $\dfrac{3}{4} + \dfrac{5}{6}$

2. $\dfrac{7}{8} - \dfrac{5}{12}$

Completely factor the following. See sections 4–2 and 4–3.

3. $2x^2 - 50$

4. $x^2 + 9x - 22$

5. $x^2 + 8x + 16$

Solve the following proportions. See section 5–4.

6. $\dfrac{3}{x} = \dfrac{5}{8}$

7. $\dfrac{5}{9} = \dfrac{y}{27}$

8. Write the number 0.0000789 in scientific notation. See section 3–5.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6–2 ■ Addition and subtraction of rational expressions

Recall that to add or subtract fractions having the same denominator, we add, or subtract, the numerators and place this sum, or difference, over the same denominator.

Addition and subtraction properties for fractions

If a, b, and c are real numbers, $b \neq 0$, then

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

■ Example 6–2 A

Add or subtract as indicated.

1. $\dfrac{3}{11} + \dfrac{4}{11} = \dfrac{3+4}{11}$ Add numerators

$= \dfrac{7}{11}$ $3 + 4 = 7$

2. $\dfrac{3}{7} - \dfrac{1}{7} = \dfrac{3-1}{7}$ Subtract numerators

$= \dfrac{2}{7}$ $3 - 1 = 2$ ■

We use the following similar procedure to add or subtract rational expressions.

Addition and subtraction properties for rational expressions

If $\dfrac{P}{R}$ and $\dfrac{Q}{R}$ are rational expressions, $R \neq 0$, then

$$\frac{P}{R} + \frac{Q}{R} = \frac{P+Q}{R} \quad \text{and} \quad \frac{P}{R} - \frac{Q}{R} = \frac{P-Q}{R}$$

Note Rational expressions having common denominators are called *like* rational expressions.

Addition and subtraction of like rational expressions

1. Add or subtract the numerators.
2. Place the sum or difference over the common denominator.
3. Reduce the resulting rational expression to lowest terms.

■ Example 6–2 B

Find the indicated sum or difference. Assume all denominators are nonzero.

1. $\dfrac{3}{x-2} + \dfrac{5}{x-2} = \dfrac{3+5}{x-2}$ Add numerators and place over $x - 2$

$= \dfrac{8}{x-2}$ $3 + 5 = 8$

2. $\dfrac{5y}{3y+5} - \dfrac{9y}{3y+5} = \dfrac{5y-9y}{3y+5}$ Subtract numerators and place over $3y+5$

$= \dfrac{-4y}{3y+5}$ $5y - 9y = -4y$

3. $\dfrac{2x-1}{x^2+5x+6} - \dfrac{4-x}{x^2+5x+6}$

$= \dfrac{(2x-1)-(4-x)}{x^2+5x+6}$ Place numerators in parentheses and subtract

$= \dfrac{2x-1-4+x}{x^2+5x+6}$ Remove parentheses and subtract

$= \dfrac{3x-5}{x^2+5x+6}$ Combine like terms

Note Notice that when we subtracted $4-x$ from $2x-1$, we placed parentheses around each polynomial. This step is *important* to avoid the common mistake of failing to change signs in the second expression when subtraction is involved.

4. $\dfrac{2x-1}{x^2+5x+6} + \dfrac{4-x}{x^2+5x+6}$

$= \dfrac{(2x-1)+(4-x)}{x^2+5x+6}$ Place numerators in parentheses and add

$= \dfrac{x+3}{x^2+5x+6}$ Remove parentheses and combine like terms

$= \dfrac{x+3}{(x+3)(x+2)}$ Factor denominator

$= \dfrac{1}{x+2}$ Reduce by common factor $x+3$

Note In the last step, *always* look for a possible reduction to lowest terms as we did in example 4.

▶ ***Quick check*** $\dfrac{4m-5}{m^2+9} - \dfrac{2m-3}{m^2+9}$ ■

When one denominator is the opposite of the other, as in the indicated sum

$$\frac{2x}{3} + \frac{5}{-3},$$

where 3 and -3 are opposites, we first multiply one of the expressions by $\dfrac{-1}{-1}$ to obtain equivalent expressions with the same denominator.

Example 6–2 C

Find the indicated sum or difference. Assume all denominators are not zero.

1. $\dfrac{2x}{3} + \dfrac{5}{-3} = \dfrac{2x}{3} + \dfrac{-1}{-1} \cdot \dfrac{5}{-3}$ Multiply $\dfrac{5}{-3}$ by $\dfrac{-1}{-1}$

$= \dfrac{2x}{3} + \dfrac{-1(5)}{-1(-3)}$ Multiply numerators and denominators

$= \dfrac{2x}{3} + \dfrac{-5}{3}$ Common denominator of 3

$= \dfrac{2x + (-5)}{3}$ Add numerators and place over 3

$= \dfrac{2x - 5}{3}$ Definition of subtraction

2. $\dfrac{5y - 1}{y - 4} + \dfrac{2y + 3}{4 - y}$

$= \dfrac{5y - 1}{y - 4} + \dfrac{-1}{-1} \cdot \dfrac{2y + 3}{4 - y}$ Multiply $\dfrac{2y + 3}{4 - y}$ by $\dfrac{-1}{-1}$

$= \dfrac{5y - 1}{y - 4} + \dfrac{-1(2y + 3)}{-1(4 - y)}$

$= \dfrac{5y - 1}{y - 4} + \dfrac{-2y - 3}{y - 4}$ Same denominator: $-1(4 - y) = y - 4$

$= \dfrac{(5y - 1) + (-2y - 3)}{y - 4}$ Add numerators in parentheses

$= \dfrac{5y - 1 - 2y - 3}{y - 4}$ Remove parentheses

$= \dfrac{3y - 4}{y - 4}$ Combine like terms

Note We could have multiplied $\dfrac{5y - 1}{y - 4}$ by $\dfrac{-1}{-1}$. The resulting denominator would then have been $4 - y$ and the numerator would have been $4 - 3y$. We would then multiply this by $\dfrac{-1}{-1}$ to obtain the same form of the answer.

3. $\dfrac{2x + 1}{x - 5} - \dfrac{x - 4}{5 - x}$

$= \dfrac{2x + 1}{x - 5} - \dfrac{-1}{-1} \cdot \dfrac{x - 4}{5 - x}$ Multiply $\dfrac{x - 4}{5 - x}$ by $\dfrac{-1}{-1}$

$= \dfrac{2x + 1}{x - 5} - \dfrac{-1(x - 4)}{-1(5 - x)}$

$= \dfrac{2x + 1}{x - 5} - \dfrac{4 - x}{x - 5}$ Same denominator: $-1(x - 4) = 4 - x$, $-1(5 - x) = x - 5$

$= \dfrac{(2x + 1) - (4 - x)}{x - 5}$ Place "( )" around numerators and subtract

$= \dfrac{2x + 1 - 4 + x}{x - 5}$ Definition of subtraction

$= \dfrac{3x - 3}{x - 5}$ Combine like terms

▶ ***Quick check*** $\dfrac{x + 7}{x - 1} + \dfrac{3x + 1}{1 - x}$ ■

The least common denominator (LCD)

If the fractions to be added or subtracted do not have the same denominator, we must change at least one of the fractions to an equivalent fraction so the fractions do have a common denominator. There are many such numbers we could use as a common denominator. However, the most convenient denominator to use is the smallest (least) number that is exactly divisible by each of the denominators—called the **least common denominator,** denoted by LCD. For example, the least common denominator (LCD) of the two fractions

$$\frac{5}{6} \text{ and } \frac{2}{9}$$

is 18, since 18 is the smallest (least) number that is exactly divisible by both 6 and 9.

Finding the LCD of a set of denominators

1. Factor each denominator completely. Write each factorization using exponential notation.
2. List each *different* factor that appears in any one of the factorizations in step 1.
3. Raise each factor of step 2 to the *greatest* power that factor has in step 1. Form the product of these factors.

Note The LCD of two or more rational expressions is also called the least common multiple (LCM) of the denominators.

■ **Example 6–2 D**

Find the LCD of rational expressions having the given denominators.

1. 6 and 9

$$\left.\begin{aligned} 6 &= 2 \cdot 3 \\ 9 &= 3 \cdot 3 = 3^2 \end{aligned}\right\} \quad \text{Factor each denominator}$$

The different factors are 2 and 3. The greatest power of 2 is 2^1 and of 3 is 3^2. The LCD is $2^1 \cdot 3^2 = 2 \cdot 9 = 18$.

2. $16a$ and $8a^3$

$$16a = 2 \cdot 2 \cdot 2 \cdot 2 \cdot a = 2^4 \cdot a$$
$$8a^3 = 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a = 2^3 \cdot a^3$$

Since the different factors are 2 and a, the greatest power of 2 is 2^4, and the greatest power of a is a^3, the LCD is $2^4 \cdot a^3 = 16a^3$.

3. $50x^3y^2$ and $20x^2y^3$

$$50x^3y^2 = 2 \cdot 5 \cdot 5 \cdot x^3 \cdot y^2 = 2 \cdot 5^2 \cdot x^3 \cdot y^2$$
$$20x^2y^3 = 2 \cdot 2 \cdot 5 \cdot x^2 \cdot y^3 = 2^2 \cdot 5 \cdot x^2 \cdot y^3$$

The different factors are 2, 5, x, and y. Since the greatest power of 2 is 2^2, of 5 is 5^2, of x is x^3, and of y is y^3, the LCD is $2^2 \cdot 5^2 \cdot x^3 \cdot y^3 = 100x^3y^3$.

4. $x^2 + x - 12$ and $x^2 + 2x - 8$

$x^2 + x - 12 = (x + 4)(x - 3)$
$x^2 + 4x - 8 = (x + 4)(x - 2)$

The different factors are $x + 4$, $x - 3$, and $x - 2$. Each factor is carried to the first power so the LCD is $(x + 4)(x - 3)(x - 2)$.

5. $x^2 - 2x + 1$, $x^2 + 11x - 12$, and $1 - x$

$x^2 - 2x + 1 = (x - 1)^2$
$x^2 + 11x - 12 = (x - 1)(x + 12)$
$1 - x = -1(x - 1)$

The different factors are $x - 1$ and $x + 12$. Since the greatest power of $x - 1$ is $(x - 1)^2$ and of $x + 12$ is $(x + 12)^1$, the LCD is $(x - 1)^2(x + 12)$.

Note Factors $x - 1$ and $1 - x$ are opposites and we will change $1 - x$ to $-1(x - 1)$ when working with the denominators. The factor -1 is then carried along in the problem.

▶ ***Quick check*** Find the LCD for $3x - 6$, $x^2 - 4x + 4$, $x^2 - 2x$.

Mastery points

Can you

- Add and subtract like rational expressions?
- Add and subtract rational expressions having denominators that are opposites?
- Find the least common denominator (LCD) of a set of rational expressions?

Exercise 6–2

Combine the given rational expressions and reduce the answer to lowest terms. Assume all denominators are nonzero. See example 6–2 B.

Example $\dfrac{4m - 5}{m^2 + 9} - \dfrac{2m - 3}{m^2 + 9}$

Solution

$= \dfrac{(4m - 5) - (2m - 3)}{m^2 + 9}$ Place numerators in parentheses and subtract

$= \dfrac{4m - 5 - 2m + 3}{m^2 + 9}$ Remove parentheses and subtract in numerator

$= \dfrac{2m - 2}{m^2 + 9}$ Combine like terms in numerator

1. $\dfrac{5}{x} + \dfrac{3}{x}$ **2.** $\dfrac{8}{y^2} + \dfrac{10}{y^2}$ **3.** $\dfrac{9}{p} - \dfrac{2}{p}$ **4.** $\dfrac{18}{m^2} - \dfrac{5}{m^2}$

$\boxed{5.}$ $\dfrac{5x}{x+2}+\dfrac{9x}{x+2}$ 6. $\dfrac{8y}{y-1}+\dfrac{-3y}{y-1}$ 7. $\dfrac{x-1}{2x}-\dfrac{x+3}{2x}$ $\boxed{8.}$ $\dfrac{3y-2}{y^2}-\dfrac{4y-1}{y^2}$

9. $\dfrac{3x+5}{x^2-1}-\dfrac{2x+3}{x^2-1}$ 10. $\dfrac{b^2+2}{b+3}-\dfrac{b^2+2b-3}{b+3}$

See example 6–2 C.

Example $\dfrac{x+7}{x-1}+\dfrac{3x+1}{1-x}$

Solution

$$=\frac{x+7}{x-1}+\frac{-1}{-1}\cdot\frac{3x+1}{1-x} \qquad \text{Multiply } \frac{3x+1}{1-x} \text{ by } \frac{-1}{-1}$$

$$=\frac{x+7}{x-1}+\frac{-1(3x+1)}{-1(1-x)}$$

$$=\frac{x+7}{x-1}+\frac{-3x-1}{x-1} \qquad -1(1-x)=x-1$$

$$=\frac{x+7-3x-1}{x-1} \qquad \text{Add numerators}$$

$$=\frac{-2x+6}{x-1} \qquad \text{Combine like terms}$$

11. $\dfrac{5}{7}+\dfrac{6}{-7}$ 12. $\dfrac{9}{10}-\dfrac{3}{-10}$ 13. $\dfrac{4}{z}-\dfrac{5}{-z}$

14. $\dfrac{6}{y}+\dfrac{9}{-y}$ 15. $\dfrac{5}{x-2}+\dfrac{12}{2-x}$ $\boxed{16.}$ $\dfrac{1}{x-7}-\dfrac{5}{7-x}$

17. $\dfrac{5y}{y-6}-\dfrac{4y}{6-y}$ 18. $\dfrac{4z}{z-3}+\dfrac{z}{3-z}$ 19. $\dfrac{x+1}{x-5}+\dfrac{2x-3}{5-x}$

20. $\dfrac{4y+3}{y-9}-\dfrac{2y-7}{9-y}$ $\boxed{21.}$ $\dfrac{2y-5}{2y-3}-\dfrac{y+7}{3-2y}$ 22. $\dfrac{z+5}{4z-3}+\dfrac{4z-1}{3-4z}$

23. $\dfrac{2x+5}{5-2x}+\dfrac{x+9}{2x-5}$ 24. $\dfrac{5-y}{6-5y}-\dfrac{9y+1}{5y-6}$

Find the least common denominator (LCD) of rational expressions having the following denominators.
See example 6–2 D.

Example $3x-6$, x^2-4x+4, and x^2-2x

Solution

$$\left.\begin{aligned} 3x-6 &= 3(x-2)\\ x^2-4x+4 &= (x-2)^2\\ x^2-2x &= x(x-2)\end{aligned}\right\} \quad \text{Factor each denominator}$$

LCD is $3x(x-2)^2$

25. $6x$ and $9x$ 26. $8a$ and $12a$ 27. $16x^2$ and $24x$

28. $6b^2$ and $14b$ 29. $28y^2$ and $35y^3$ 30. $9z^3$ and $7z^4$

31. $32a^2$ and $64a^4$ $\boxed{32.}$ $4x^2$, $3x$, and $8x^3$ 33. $10a^2$, $12a^3$, and $9a$

34. $4x-2$ and $2x-1$ 35. $x-4$ and $3x-12$ 36. $6x-12$ and $9x-18$

37. $18y^3$ and $9y - 36$

38. $32z^2$ and $16z - 32$

39. $a^2 + a$ and $a^2 - 1$

40. $(z - 1)^2$ and $z^2 - 1$

41. $8a + 16$ and $a^2 + 3a + 2$

42. $9p - 18$ and $p^2 - 7p + 10$

43. $a^2 - 5a + 6$ and $a^2 - 4$

44. $y^2 - y - 12$ and $y^2 + 6y + 9$

45. $a^2 - 9$ and $a^2 - 5a + 6$

46. $p^2 - 9$, $p^2 + p - 6$, and $p^2 - 4p + 4$

47. $x^2 - 49$, $7 - x$, and $2x + 14$

48. $5 - y$, $y^2 - 25$, and $y^2 - 10y + 25$

Review exercises

1. The statement $4(y + 3) = 4(3 + y)$ demonstrates what property of real numbers? See section 1–8.

Factor the following expressions. See sections 4–2, 4–3, and 4–4.

2. $5y^2 - 20$

3. $x^2 + 20x + 100$

4. $3y^2 - y - 4$

Find the solution set of the following equations. See sections 2–6 and 4–7.

5. $4(x + 3) = 5(4 - 3x)$

6. $x^2 - 2x = 15$

6–3 ■ Addition and subtraction of rational expressions

Now that we can find the least common denominator (LCD) of a group of rational expressions, let us review the process for changing a fraction (or rational expression) to an equivalent fraction with a new denominator.

■ Example 6–3 A

1. Change $\frac{7}{15}$ to an equivalent fraction having denominator 60.

We want $\frac{7}{15} = \frac{?}{60}$.

Since $60 = 15 \cdot 4$ (factor 4 is found by dividing $60 \div 15 = 4$), we multiply the given fraction by $\frac{4}{4}$ $\left(\frac{4}{4} = 1\right)$.

$$\begin{aligned}\frac{7}{15} &= \frac{7}{15} \cdot \frac{4}{4} \\ &= \frac{7 \cdot 4}{15 \cdot 4} && \text{Multiply numerators} \\ & && \text{Multiply denominators} \\ &= \frac{28}{60}\end{aligned}$$

Thus, $\frac{7}{15} = \frac{28}{60}$.

2. Change $\dfrac{x+1}{x-4}$ to an equivalent rational expression having denominator $x^2 - 2x - 8$.

We want $\dfrac{x+1}{x-4} = \dfrac{?}{x^2 - 2x - 8}$.

Since $x^2 - 2x - 8 = (x-4)(x+2)$, we multiply the given rational expression by $\dfrac{x+2}{x+2}$.

$$\begin{aligned}\frac{x+1}{x-4} &= \frac{x+1}{x-4} \cdot \frac{x+2}{x+2} \\ &= \frac{(x+1)(x+2)}{(x-4)(x+2)} && \text{Multiply numerators} \\ & && \text{Multiply denominators} \\ &= \frac{x^2+3x+2}{x^2-2x-8} && \text{Perform indicated operations}\end{aligned}$$

Thus, $\dfrac{x+1}{x-4} = \dfrac{x^2+3x+2}{x^2-2x-8}$. ■

Once equivalent rational expressions are obtained with the LCD as the denominator, we add or subtract as previously learned. Use the following steps to add or subtract rational expressions having unlike denominators.

Addition and subtraction of rational expressions having different denominators

1. Find the LCD of the rational expressions.
2. Write each rational expression as an equivalent rational expression with the LCD as the denominator.
3. Perform the indicated addition or subtraction as before.
4. Reduce the results to lowest terms.

■ Example 6–3 B

Add the following rational expressions. Assume the denominators are not equal to zero. Reduce all answers to lowest terms.

1. $\dfrac{5x}{8} + \dfrac{7x}{12}$

$$\left.\begin{aligned} 8 &= 2 \cdot 2 \cdot 2 \\ 12 &= 2 \cdot 2 \cdot 3 \end{aligned}\right\} \text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 = 24 \qquad \text{Find the LCD}$$

Since $\dfrac{24}{8} = 3$ and $\dfrac{24}{12} = 2$

$$\begin{aligned}\frac{5x}{8} + \frac{7x}{12} &= \frac{5x}{8} \cdot \frac{3}{3} + \frac{7x}{12} \cdot \frac{2}{2} && \text{Multiply } \frac{5x}{8} \text{ by } \frac{3}{3} \text{ and } \frac{7x}{12} \text{ by } \frac{2}{2} \\ &= \frac{15x}{24} + \frac{14x}{24} && \text{Multiply numerators and denominators} \\ &= \frac{15x + 14x}{24} && \text{Add numerators} \\ &= \frac{29x}{24} && \text{Combine as indicated}\end{aligned}$$

2. $\dfrac{15}{4x^2} + \dfrac{25}{18x}$

$$\left.\begin{aligned} 4x^2 &= 2 \cdot 2 \cdot x^2 \\ 18x &= 2 \cdot 3 \cdot 3 \cdot x \end{aligned}\right\} \text{LCD} = 2 \cdot 2 \cdot 3 \cdot 3 \cdot x^2 = 36x^2$$ Find the LCD

Since $\dfrac{36x^2}{4x^2} = 9$ and $\dfrac{36x^2}{18x} = 2x$

$$\frac{15}{4x^2} + \frac{25}{18x} = \frac{15}{4x^2} \cdot \frac{9}{9} + \frac{25}{18x} \cdot \frac{2x}{2x}$$ Multiply $\frac{15}{4x^2}$ by $\frac{9}{9}$ and $\frac{25}{18x}$ by $\frac{2x}{2x}$

$$= \frac{135}{36x^2} + \frac{50x}{36x^2}$$ Multiply numerators and denominators

$$= \frac{135 + 50x}{36x^2}$$ Add numerators

3. $\dfrac{3y + 2}{y^2 - 16} + \dfrac{y - 4}{3y + 12}$

$$\left.\begin{aligned} y^2 - 16 &= (y + 4)(y - 4) \\ 3y + 12 &= 3(y + 4) \end{aligned}\right\} \text{LCD} = 3(y + 4)(y - 4)$$ Find the LCD

Since $\dfrac{3(y + 4)(y - 4)}{(y + 4)(y - 4)} = 3$ and $\dfrac{3(y + 4)(y - 4)}{3(y + 4)} = y - 4$, then

$$\frac{3y + 2}{y^2 - 16} + \frac{y - 4}{3y + 12}$$
$$= \frac{3y + 2}{(y + 4)(y - 4)} \cdot \frac{3}{3} + \frac{y - 4}{3(y + 4)} \cdot \frac{y - 4}{y - 4}$$ Multiply $\frac{3y + 2}{(y + 4)(y - 4)}$ by $\frac{3}{3}$ and $\frac{y - 4}{3(y + 4)}$ by $\frac{y - 4}{y - 4}$

$$= \frac{3(3y + 2)}{3(y + 4)(y - 4)} + \frac{(y - 4)(y - 4)}{3(y + 4)(y - 4)}$$ Multiply numerators and denominators

$$= \frac{(9y + 6) + (y^2 - 8y + 16)}{3(y + 4)(y - 4)}$$ Add numerators

$$= \frac{y^2 + y + 22}{3(y + 4)(y - 4)}$$ Remove parentheses and combine

Note When the numerators have two or more terms as in example 3, we place the quantities in parentheses when we add the numerators. This is a good practice to avoid a *most common* mistake when subtracting.

▶ ***Quick check*** Add the rational expressions $\dfrac{20}{3y} + \dfrac{25}{12y^2}$. ■

Example 6–3 C

Subtract the following rational expressions. Assume the denominators are not equal to zero. Reduce to lowest terms.

1. $\dfrac{5y}{2y-1} - \dfrac{y+1}{y+2}$

The LCD of the rational expressions is $(2y-1)(y+2)$.

$$\frac{5y}{2y-1} - \frac{y+1}{y+2}$$

$$= \frac{5y}{2y-1} \cdot \frac{y+2}{y+2} - \frac{y+1}{y+2} \cdot \frac{2y-1}{2y-1}$$

Multiply $\dfrac{5y}{2y-1}$ by $\dfrac{y+2}{y+2}$ and $\dfrac{y+1}{y+2}$ by $\dfrac{2y-1}{2y-1}$

$$= \frac{5y(y+2)}{(2y-1)(y+2)} - \frac{(y+1)(2y-1)}{(2y-1)(y+2)}$$

Multiply numerators and denominators

$$= \frac{(5y^2+10y) - (2y^2+y-1)}{(2y-1)(y+2)}$$

Subtract numerators

$$= \frac{5y^2+10y-2y^2-y+1}{(2y-1)(y+2)}$$

Remove parentheses and change signs when subtracting

$$= \frac{3y^2+9y+1}{(2y-1)(y+2)}$$

Combine like terms

Don't forget the parentheses

Note The numerator $3y^2 + 9y + 1$ cannot be factored so we are unable to reduce. We should always check this!

2. $\dfrac{5x-4}{x^2-2x+1} - \dfrac{3x}{x^2+4x-5}$

$$\left.\begin{aligned} x^2-2x+1 &= (x-1)^2 \\ x^2+4x-5 &= (x+5)(x-1) \end{aligned}\right\} \text{LCD} = (x-1)^2(x+5)$$

Find the LCD

$$\frac{5x-4}{x^2-2x+1} - \frac{3x}{x^2+4x-5}$$

Factor denominators

$$= \frac{5x-4}{(x-1)^2} - \frac{3x}{(x+5)(x-1)}$$

$$= \frac{5x-4}{(x-1)^2} \cdot \frac{x+5}{x+5} - \frac{3x}{(x+5)(x-1)} \cdot \frac{x-1}{x-1}$$

Multiply $\dfrac{5x-4}{(x-1)^2}$ by $\dfrac{x+5}{x+5}$ and $\dfrac{3x}{(x+5)(x-1)}$ by $\dfrac{x-1}{x-1}$

$$= \frac{(5x-4)(x+5)}{(x-1)^2(x+5)} - \frac{3x(x-1)}{(x-1)^2(x+5)}$$

Multiply numerators and denominators

$$= \frac{(5x^2+21x-20) - (3x^2-3x)}{(x-1)^2(x+5)}$$

Subtract numerators

$$= \frac{5x^2+21x-20-3x^2+3x}{(x-1)^2(x+5)}$$

Remove parentheses and change signs

$$= \frac{2x^2+24x-20}{(x-1)^2(x+5)}$$

Combine like terms

Don't forget the parentheses

▶ ***Quick check*** Subtract $\dfrac{y+1}{y^2-y-12} - \dfrac{3y+2}{y^2-9y+20}$ ■

Problem solving

■ Example 6–3 D

Set up a rational expression for the following word statements.

1. If Dick can mow his lawn in h hours, what part of the lawn can he mow in 2 hours?

We must find what part of the total time, h hours, is 2 hours. Thus, Dick can mow the fractional part

$$\frac{2}{h}$$

of the lawn in 2 hours.

2. If the area (A) of a rectangle is n square inches, what is the expression for the length, ℓ, if the width, w, is 12 inches? ($A = \ell \cdot w$)

Using $A = \ell \cdot w$, we have $\ell = \dfrac{A}{w}$ and so the length

$\ell = \dfrac{n}{12}$ inches. Replace A with n and w with 12

▶ ***Quick check*** If the area of a rectangle is p square yards, write an expression for the width, w, if the length is 5 yards. ■

Mastery points

Can you

■ Add and subtract rational expressions having unlike denominators?

Exercise 6–3

Perform the indicated addition and reduce the answer to lowest terms. Assume all denominators are not equal to zero. See example 6–3 B.

Example $\dfrac{20}{3y} + \dfrac{25}{12y^2}$

Solution $\left.\begin{array}{l} 3y = 3 \cdot y \\ 12y^2 = 3 \cdot 2 \cdot 2 \cdot y \cdot y \end{array}\right\}$ LCD $= 3 \cdot 2 \cdot 2 \cdot y \cdot y = 12y^2$

$\dfrac{12y^2}{3y} = 4y$ and the denominator of the second expression is the LCD, $12y^2$.

$$\frac{20}{3y} + \frac{25}{12y^2} = \frac{20}{3y} \cdot \frac{4y}{4y} + \frac{25}{12y^2}$$ Multiply numerator and denominator of $\frac{20}{3y}$ by $4y$

$$= \frac{80y}{12y^2} + \frac{25}{12y^2}$$ Perform indicated operations

$$= \frac{80y + 25}{12y^2}$$ Add numerators and place over common denominator

1. $\frac{x}{6} + \frac{3}{4}$
2. $\frac{3z}{10} + \frac{2z}{15}$
3. $\frac{2x-1}{16} + \frac{x+2}{24}$
4. $\frac{4}{3x} + \frac{5}{2x}$
5. $\frac{3a+1}{a} + \frac{2a-3}{3a}$
6. $\frac{4}{x-1} + \frac{5}{x+3}$
7. $\frac{8}{y+4} + \frac{7}{y-5}$
8. $\frac{x}{x+2} + \frac{3x}{4x-1}$
9. $\frac{15}{5y-10} + \frac{14}{2y-4}$
10. $\frac{21}{6x+12} + \frac{15}{2x+4}$
11. $\frac{12}{x^2-4} + \frac{7}{4x-8}$
12. $\frac{16}{2y+6} + \frac{5}{y^2-9}$
13. $5 + \frac{4x}{x+8}$
14. $9 + \frac{y+9}{y-1}$
15. $\frac{x}{x-1} + \frac{3x}{x^2-1}$
16. $\frac{2y}{y^2-16} + \frac{5y}{2y-8}$
17. $\frac{4}{x^2-x-6} + \frac{5}{x^2-9}$
18. $\frac{6}{x^2-4x-12} + \frac{5}{x^2-36}$
19. $\frac{2y}{y^2-6y+9} + \frac{5y}{y^2-2y-3}$
20. $\frac{4z}{z^2+z-20} + \frac{z}{z^2-8z+16}$
21. $\frac{y-2}{y^2-3y-10} + \frac{y+1}{y^2-y-6}$
22. $\frac{2x+1}{x^2+6x+5} + \frac{4x-3}{x^2-x-30}$

Perform the indicated subtraction and reduce to lowest terms. Assume all denominators are not zero. See example 6–3 C.

Example $\frac{y+1}{y^2-y-12} - \frac{3y+2}{y^2-9y+20}$

Solution $\left.\begin{aligned} y^2-y-12 &= (y-4)(y+3) \\ y^2-9y+20 &= (y-4)(y-5) \end{aligned}\right\}$ LCD is $(y-4)(y+3)(y-5)$.

$$\frac{y+1}{y^2-y-12} - \frac{3y+2}{y^2-9y+20}$$

$$= \frac{y+1}{(y-4)(y+3)} \cdot \frac{y-5}{y-5} - \frac{3y+2}{(y-4)(y-5)} \cdot \frac{y+3}{y+3}$$

Multiply numerator and denominator of $\frac{y+1}{(y-4)(y+3)}$ by $y-5$ and of $\frac{3y+2}{(y-4)(y-5)}$ by $y+3$

$$= \frac{(y+1)(y-5)}{(y-4)(y+3)(y-5)} - \frac{(3y+2)(y+3)}{(y-4)(y+3)(y-5)}$$

Multiply numerators and denominators

$$= \frac{(y^2-4y-5)-(3y^2+11y+6)}{(y-4)(y+3)(y-5)}$$

Multiply in each numerator, place parentheses around each product, and subtract

$$= \frac{y^2-4y-5-3y^2-11y-6}{(y-4)(y+3)(y-5)}$$

Remove parentheses, change signs, and subtract

$$= \frac{-2y^2-15y-11}{(y-4)(y+3)(y-5)}$$

Combine like terms in numerator

Don't forget the parentheses

Note The denominator is left in factored form in case the answer can be reduced.

23. $\frac{y}{9} - \frac{5}{6}$

24. $\frac{5y}{12} - \frac{y}{8}$

25. $\frac{9}{14y} - \frac{1}{21y}$

26. $\frac{7}{12z} - \frac{10}{9z}$

27. $\frac{5a + 3}{12} - \frac{a - 4}{10}$

28. $\frac{2x + 9}{8} - \frac{x - 7}{20}$

29. $\frac{2x + 5}{6x} - \frac{x - 5}{9x}$

30. $\frac{4y - 1}{3y} - \frac{2y - 3}{15y}$

31. $\frac{7}{2x - 3} - \frac{6}{x - 5}$

32. $\frac{7}{4x - 6} - \frac{12}{3x + 9}$

33. $\frac{12}{3y + 6} - \frac{11}{7y + 14}$

34. $\frac{14}{5x - 15} - \frac{8}{2x - 6}$

35. $9 - \frac{6}{x + 8}$

36. $12 - \frac{7}{z - 12}$

37. $\frac{2x}{3x + 1} - 9$

38. $\frac{4y}{5y - 4} - 10$

39. $\frac{-3}{a^2 - 5a + 6} - \frac{3}{a^2 - 4}$

40. $\frac{8}{x^2 - 25} - \frac{7}{x^2 + 3x - 10}$

41. $\frac{20}{y^2 - 2y - 24} - \frac{8}{y^2 + y - 12}$

42. $\frac{2p}{p^2 - 9p + 20} - \frac{5p - 2}{p - 5}$

43. $\frac{2a - 3}{a^2 - 5a + 6} - \frac{3a}{a - 2}$

Add and subtract as indicated.

44. $\frac{13}{12b} - \frac{2}{9b} + \frac{5}{4b}$

45. $\frac{5}{6z} + \frac{4}{8z} - \frac{1}{4z}$

46. $\frac{3x}{8} - \frac{2x}{5} + \frac{7x}{10}$

47. $\frac{4a}{5} + \frac{7a}{15} - \frac{a}{9}$

48. $\frac{a - 1}{5} - \frac{2a + 3}{15} + \frac{3a - 1}{25}$

49. $\frac{5b + 1}{6} + \frac{3b - 2}{9} - \frac{b + 1}{12}$

50. $\frac{4a}{a^2 + 2a - 15} + \frac{3a}{2a^2 + 11a + 5} - \frac{5a}{2a^2 - 5a - 3}$

51. $\frac{5}{z^2 - 4} - \frac{z}{z^2 - 1} + \frac{4}{z^2 + z - 2}$

Solve the following word problems.

52. For a lens maker to manufacture lenses that will refract light by exactly the right amounts, the following formula is used:

$$\frac{1}{f} = (u - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Add the expressions containing R_1 and R_2.

53. Women A, B, and C can complete a given job in a, b, and c hours, respectively. Working together they can complete in one hour $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ of the job. By combining, obtain a single expression for what they can do together in one hour.

54. In electricity, the total resistance of any parallel circuit may be given by

$$\frac{1}{R_t} = \frac{I_1}{E_1} + \frac{I_2}{E_2} + \frac{I_3}{E_3}$$

Combine the expression in the right member.

See example 6–3 D.

Example If the area of a rectangle is p square yards, write an expression for the width, w, if the length is 5 yards.

Solution Using $A = \ell \cdot w$, we are given that $A = p$ square yards and $\ell = 5$ yards. Substituting, we obtain

$$p = 5 \cdot w$$

and solving for w, we divide each member by 5. Thus $w = \dfrac{p}{5}$.

55. A faucet when fully open, can fill the sink in m minutes. What part of the sink can it fill in 3 minutes?

56. An inlet pipe to a swimming pool can fill the pool in h hours. What part of the pool can it fill in 9 hours?

57. An outlet pipe can drain a swimming pool in 36 hours. What part of the pool can it drain in h hours?

58. Jane can paint her house in h hours. What part of the house can she paint in 1 hour?

59. The product of two numbers is 48. If one of the numbers is m, what is the other number?

60. The area of a rectangle is 54 square centimeters. What is the length ℓ if the width is w centimeters?

61. The area of a rectangle is A square feet. What is the width if the length is 23 feet?

62. The area of a triangle is 21 square yards. If the triangle has a base length b, what is the altitude h of the triangle? $\left(A = \dfrac{1}{2}bh\right)$

63. The area of a triangle is A square rods. If the altitude h is 9 rods, what is the length of the base b? (See problem 62.)

64. John drives 25 miles in h hours. At what speed, r, did he travel? [*Hint:* Use distance traveled (d) = rate $(r) \times$ time (t).]

65. Mabel travels d miles at a rate of 55 miles per hour. Write an expression for the time t that she traveled. (See problem 64.)

66. What is the reciprocal of the natural number n?

67. What is the reciprocal of the fraction $\dfrac{a}{b}$?

Review exercises

Completely factor the following expressions. See sections 4–2, 4–3, and 4–4.

1. $x^2 - 14x + 49$ **2.** $2x^2 - 11x + 5$ **3.** $4x^2 - 16$

Multiply the following expressions. See section 3–2.

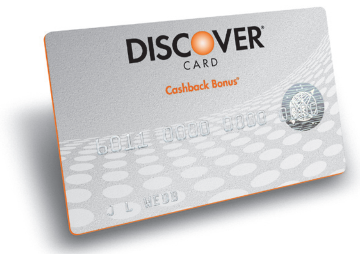
4. $(x + 9)(x - 9)$ **5.** $(4x + 3)^2$ **6.** $(2x + 1)(x - 8)$

Find the LCD of expressions having the following denominators. See section 6–2.

7. 16, 12, 6 **8.** $4x, 2x^2, 6$ **9.** $x^2 - 9; (x - 3)^2; x + 3$

Add or subtract the following. See section 6–3.

10. $\dfrac{5}{x} + \dfrac{3}{2x}$ **11.** $\dfrac{5}{x - 2} - \dfrac{3}{x - 1}$



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6–4 ■ Complex fractions

A complex fraction

A **complex fraction** is a fraction (rational expression) whose numerator, denominator, or both contain fractions (or rational expressions). The expressions

$$\frac{\frac{3}{4}}{\frac{5}{6}}, \quad \frac{\frac{1}{3}+2}{1-\frac{1}{2}}, \quad \text{and} \quad \frac{\frac{x-1}{x-2}}{\frac{x+3}{x}}$$

are examples of complex fractions. Given a complex fraction, we simplify the fraction by *eliminating the fractions within the numerator and/or the denominator* to obtain a simple fraction.

We name the parts of a complex fraction as shown in the following examples.

$$\frac{\left.\frac{3}{x}+\frac{4}{y}\right\}\text{ Primary numerator}}{\left.\frac{1}{x}-\frac{2}{y}\right\}\text{ Primary denominator}}$$

$$\frac{\frac{3}{x}+\frac{4}{y}}{\frac{1}{x}-\frac{2}{y}}$$

Secondary numerators (3, 4)
Secondary denominators (x, y)
Secondary numerators (1, 2)
Secondary denominators (x, y)

To simplify a complex fraction, we use one of the following methods.

> **Simplifying a complex fraction**
>
> ***Method 1*** Multiply the primary numerator and the primary denominator by the LCD of the secondary denominators and reduce if possible.
>
> ***Method 2*** Form a single fraction in the numerator and in the denominator and divide the primary numerator by the primary denominator.

■ Example 6–4 A

Simplify each complex fraction.

1. $\dfrac{\frac{3}{4}}{\frac{5}{6}}$

Method 1

$$\frac{\frac{3}{4}}{\frac{5}{6}} = \frac{\frac{3}{4}\cdot 12}{\frac{5}{6}\cdot 12}$$ Multiply primary numerator and primary denominator by the LCD of secondary denominators 4 and 6, which is 12

$$= \frac{3\cdot\frac{12}{4}}{5\cdot\frac{12}{6}}$$ Divide 4 into 12; Divide 6 into 12

$$= \frac{3\cdot 3}{5\cdot 2}$$ Multiply in numerator and denominator

$$= \frac{9}{10}$$

Method 2

$$\frac{\frac{3}{4}}{\frac{5}{6}} = \frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \cdot \frac{6}{5}$$ Multiply by the reciprocal of $\frac{5}{6}$

$$= \frac{3 \cdot 6}{4 \cdot 5}$$ Multiply numerators and denominators as indicated

$$= \frac{18}{20}$$ Multiply and reduce to lowest terms

$$= \frac{9}{10}$$ Reduce to lowest terms

2. $$\frac{\frac{3}{a} - 1}{1 + \frac{4}{b}}$$

Method 1

The LCD of the secondary denominators, a and b, is ab. We multiply the primary numerator, $\frac{3}{a} - 1$, and the primary denominator, $1 + \frac{4}{b}$, by ab. Remember to multiply the LCD times *every term* in the numerator and the denominator.

$$\frac{\frac{3}{a} - 1}{1 + \frac{4}{b}} = \frac{\left(\frac{3}{a} - 1\right) \cdot ab}{\left(1 + \frac{4}{b}\right) \cdot ab}$$

$$= \frac{\frac{3}{a} \cdot ab - 1 \cdot ab}{1 \cdot ab + \frac{4}{b} \cdot ab}$$ Multiply each term of numerator and denominator by ab

$$= \frac{3b - ab}{ab + 4a}$$ Perform indicated multiplications

$$= \frac{b(3 - a)}{a(b + 4)}$$ Factor numerator and denominator

Method 2

We first change the primary numerator, $\frac{3}{a} - 1$, and the primary denominator, $1 + \frac{4}{b}$, to single fractions. Thus,

$$\frac{3}{a} - 1 = \frac{3}{a} - \frac{a}{a} = \frac{3 - a}{a}$$ Combine in primary numerator

and

$$1 + \frac{4}{b} = \frac{b}{b} + \frac{4}{b} = \frac{b + 4}{b}$$ Combine in primary denominator

$$\frac{\frac{3}{a}-1}{1+\frac{4}{b}}=\frac{\frac{3-a}{a}}{\frac{b+4}{b}}=\frac{3-a}{a}\cdot\frac{b}{b+4}$$ Multiply by the reciprocal of $\frac{b+4}{b}$

$$=\frac{b(3-a)}{a(b+4)}$$

Note Always form a single fraction in the numerator and in the denominator *before* we invert and multiply. That is,

$$\frac{\frac{3}{a}-1}{1+\frac{4}{b}}\neq\left(\frac{3}{a}-1\right)\cdot\left(1+\frac{b}{4}\right)$$

As you can see, both methods will work. We will use only Method 1 in the remaining examples. Choice of method is left up to the student.

3. $\dfrac{\frac{4}{x}-\frac{3}{y}}{\frac{5}{x}+\frac{7}{y}}$

The LCD of the secondary denominators, x and y, is xy. We multiply the primary numerator, $\frac{4}{x}-\frac{3}{y}$, and the primary denominator, $\frac{5}{x}+\frac{7}{y}$, by xy and get

$$\frac{\frac{4}{x}-\frac{3}{y}}{\frac{5}{x}+\frac{7}{y}}=\frac{\left(\frac{4}{x}-\frac{3}{y}\right)\cdot xy}{\left(\frac{5}{x}+\frac{7}{y}\right)\cdot xy}=\frac{\frac{4}{x}\cdot xy-\frac{3}{y}\cdot xy}{\frac{5}{x}\cdot xy+\frac{7}{y}\cdot xy}$$ Multiply each term of numerator and denominator by xy

$$=\frac{4y-3x}{5y+7x}$$ Perform indicated multiplications

4. $\dfrac{\frac{a^2-x^2}{x}}{\frac{a+x}{x^2}}$

The LCD of the secondary denominators, x and x^2, is x^2. We then multiply the primary numerator, $\frac{a^2-x^2}{x}$, and the primary denominator, $\frac{a+x}{x^2}$, by x^2 and get

$$\frac{\frac{a^2-x^2}{x}}{\frac{a+x}{x^2}}=\frac{\left(\frac{a^2-x^2}{x}\right)\cdot x^2}{\left(\frac{a+x}{x^2}\right)\cdot x^2}=\frac{(a^2-x^2)\cdot x}{a+x}$$

$$=\frac{(a+x)(a-x)\cdot x}{a+x}$$ Factor the numerator

$$=(a-x)\cdot x$$ Reduce by the common factor $a+x$

$$=ax-x^2$$ Multiply as indicated

Note In example 4, we proceeded to *reduce before* performing the multiplication in the numerator. *Always do this, if possible.*

▶ ***Quick check*** Simplify $\dfrac{\dfrac{3}{4}+\dfrac{2}{3}}{\dfrac{5}{6}-\dfrac{1}{12}}$

Mastery points

Can you

- Simplify complex fractions?

Exercise 6–4

Simplify each complex fraction to a simple fraction. See example 6–4 A.

Example $\dfrac{\dfrac{3}{4}+\dfrac{2}{3}}{\dfrac{5}{6}-\dfrac{1}{12}}$

Solution *Method 1*

$$\frac{\dfrac{3}{4}+\dfrac{2}{3}}{\dfrac{5}{6}-\dfrac{1}{12}}=\frac{\left(\dfrac{3}{4}+\dfrac{2}{3}\right)12}{\left(\dfrac{5}{6}-\dfrac{1}{12}\right)12}=\frac{\dfrac{3}{4}\cdot 12+\dfrac{2}{3}\cdot 12}{\dfrac{5}{6}\cdot 12-\dfrac{1}{12}\cdot 12}$$ Multiply numerator and denominator by 12

$$=\frac{9+8}{10-1}$$ Multiply as indicated

$$=\frac{17}{9}$$ Combine in the numerator and the denominator

Method 2

$$\frac{\dfrac{3}{4}+\dfrac{2}{3}}{\dfrac{5}{6}-\dfrac{1}{12}}=\frac{\dfrac{9}{12}+\dfrac{8}{12}}{\dfrac{10}{12}-\dfrac{1}{12}}=\frac{\dfrac{17}{12}}{\dfrac{9}{12}}$$ Add in the numerator

Subtract in the denominator

$$=\frac{17}{12}\div\frac{9}{12}$$

$$=\frac{17}{12}\cdot\frac{12}{9}$$ Multiply by the reciprocal of $\frac{9}{12}$

$$=\frac{17}{9}$$

1. $\dfrac{\dfrac{2}{3}}{\dfrac{4}{5}}$ **2.** $\dfrac{\dfrac{7}{8}}{\dfrac{5}{6}}$ **3.** $\dfrac{\dfrac{4}{3}}{\dfrac{8}{9}}$ **4.** $\dfrac{\dfrac{9}{10}}{\dfrac{7}{6}}$ **5.** $\dfrac{1+\dfrac{3}{5}}{2-\dfrac{1}{5}}$

6. $\dfrac{5-\frac{3}{4}}{1+\frac{5}{8}}$

$\boxed{7.}$ $\dfrac{7}{2+\frac{4}{5}}$

8. $\dfrac{10}{4-\frac{11}{12}}$

9. $\dfrac{4+\frac{3}{5}}{6}$

10. $\dfrac{10-\frac{7}{8}}{3}$

11. $\dfrac{\frac{3}{4}+\frac{5}{8}}{\frac{1}{2}-\frac{1}{4}}$

$\boxed{12.}$ $\dfrac{\frac{6}{7}-\frac{5}{14}}{\frac{3}{14}-\frac{5}{7}}$

13. $\dfrac{x+\frac{1}{4}}{x-\frac{3}{4}}$

14. $\dfrac{y-\frac{5}{6}}{y+\frac{1}{2}}$

$\boxed{15.}$ $\dfrac{\frac{1}{a}+3}{\frac{2}{a}-4}$

16. $\dfrac{5-\frac{3}{b}}{6+\frac{5}{b}}$

17. $\dfrac{\frac{3}{a^2}+4}{5-\frac{3}{a}}$

18. $\dfrac{\frac{5}{x}-5}{6+\frac{4}{x^3}}$

19. $\dfrac{a-\frac{3}{b}}{a+\frac{4}{b}}$

20. $\dfrac{x+\frac{4}{y}}{x-\frac{5}{y}}$

21. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}$

$\boxed{22.}$ $\dfrac{\frac{3}{x^2}-\frac{4}{y}}{\frac{5}{x}+\frac{2}{y^2}}$

23. $\dfrac{x+y}{\frac{1}{x}+\frac{1}{y}}$

24. $\dfrac{a-b}{\frac{2}{a}+\frac{3}{b}}$

25. $\dfrac{\frac{4}{a^2}-\frac{5}{b}}{a-b}$

26. $\dfrac{\frac{6}{x^2}-\frac{5}{y^2}}{x+y}$

$\boxed{27.}$ $\dfrac{\frac{1}{x+y}-\frac{1}{x-y}}{\frac{1}{x+y}+\frac{1}{x-y}}$

28. $\dfrac{\frac{a}{a+b}-\frac{b}{a-b}}{\frac{b}{a+b}+\frac{a}{a-b}}$

29. $\dfrac{\frac{x+y}{x-y}+\frac{x-y}{x+y}}{x^2-y^2}$

30. $\dfrac{\frac{b}{a+b}-\frac{a}{a-b}}{a^2-b^2}$

31. $\dfrac{\frac{2}{ab}+\frac{3}{ab^2}}{a^2b^2}$

32. $\dfrac{\frac{5}{x^2y^2}-\frac{4}{xy}}{xy}$

33. $\dfrac{\frac{2}{a+3}+\frac{1}{a-2}}{\frac{3}{a-2}-\frac{4}{a+3}}$

34. $\dfrac{\frac{7}{x-4}-\frac{5}{x+3}}{\frac{5}{x+3}+\frac{9}{x-4}}$

35. $\dfrac{\frac{5}{x^2-x-12}}{\frac{4}{x+3}-\frac{5}{x-4}}$

$\boxed{36.}$ $\dfrac{\frac{7}{b-7}+\frac{8}{b-5}}{\frac{6}{b^2-12b+35}}$

37. $\dfrac{3-\frac{4}{a+4}}{\frac{5}{a+4}-\frac{6}{a-1}}$

38. $\dfrac{\frac{6}{x-5}+7}{\frac{8}{x-5}-\frac{9}{x+3}}$

39. $\dfrac{\frac{y^2-y-6}{y^2+2y+1}}{\frac{y^2-2y-8}{y^2+7y+6}}$

40. $\dfrac{\frac{a^2+3a-10}{a^2-5a-14}}{\frac{a^2+6a+5}{a^2-8a+7}}$

Solve the following word problems.

41. A refrigeration coefficient-of-performance formula for the ideal refrigerator is given by

$$cp = \frac{1}{\frac{T_2}{T_1}-1}$$

Simplify the right member.

42. In electronics, a formula for coupled inductance in parallel with fields aiding is given by

$$L_t = \frac{1}{\frac{1}{L_1-M}+\frac{1}{L_2+M}}$$

Simplify the right member.

43. In electronics, a formula for self-inductance of circuits in parallel is given by

$$L_t = \frac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2} + \dfrac{1}{L_3}}$$

Simplify the right member.

44. Coupled inductance with circuits connected in parallel with opposing fields is given by

$$L_t = \frac{1}{\dfrac{1}{L_1 - M} + \dfrac{1}{L_2 - M}}$$

Simplify the right member.

Review exercises

Reduce the following expressions to lowest terms. See section 5–2.

1. $\dfrac{36}{42}$

2. $\dfrac{x^2 - 5x - 14}{x^2 + 4x + 4}$

3. Subtract $(3x^3 - 2x^2 + x - 12) - (x^3 - 5x^2 + 9)$. See section 2–3.

Solve the following equations. See sections 2–6 and 4–7.

4. $4x + 3x = 21$

5. $8y - 4 = 5y - 10$

6. $x^2 + 2x - 3 = 0$

Determine the domain of the following rational expressions. See section 5–1.

7. $\dfrac{3}{x + 7}$

8. $\dfrac{y - 4}{y^2 - 4}$

9. Perform the indicated operations, if possible. See section 1–7.

a. $\dfrac{4}{0}$ b. $\dfrac{0}{-3}$

6–5 ■ Rational equations

A rational equation

An algebraic equation that contains *at least one* rational expression is called a **rational equation.**

The basic operations for solving equations that you learned in chapter 2 will apply to rational equations once the denominators in the equation are eliminated. We accomplish this by using the multiplication property of equality. The multiplier is the least common denominator of all denominators in the rational expressions of the equation.

Solving a rational equation

1. Find the LCD of all denominators.
2. Eliminate the denominators by multiplying each term of both members of the equation by the LCD of the denominators in the equation.
3. Use the four steps from section 2–6 to solve the resulting equation.

Example 6–5 A

Find the solution set of each of the following rational equations.

1. $\dfrac{x-3}{4} = \dfrac{x}{8}$

We can see that the LCD of the denominators 4 and 8 is 8.

$$\frac{8}{1} \cdot \frac{(x-3)}{4} = \frac{8}{1} \cdot \frac{x}{8} \quad \text{Multiply each member by 8}$$
$$2(x-3) = x \quad \text{Reduce in each member}$$
$$2x - 6 = x \quad \text{Multiply as indicated}$$
$$x - 6 = 0 \quad \text{Subtract } x \text{ from each member}$$
$$x = 6 \quad \text{Add 6 to each member}$$

The solution set is $\{6\}$. If we wish to check our work, we substitute 6 for x in the original equation to obtain the equivalent equation.

$$\frac{6-3}{4} = \frac{6}{8} \quad \text{Replace } x \text{ with 6 in the original equation}$$
$$\frac{3}{4} = \frac{3}{4} \quad \text{(True)}$$

2. $\dfrac{t}{4} - \dfrac{t-4}{5} = \dfrac{7}{10}$

The LCD of the denominators 4, 5, and 10 is 20.

$$\frac{20}{1} \cdot \frac{t}{4} - \frac{20}{1} \cdot \frac{t-4}{5} = \frac{20}{1} \cdot \frac{7}{10} \quad \text{Multiply each term by the LCD 20}$$
$$5t - 4(t-4) = 2 \cdot 7 \quad \text{Reduce each term}$$
$$5t - 4t + 16 = 14 \quad \text{Multiply as indicated}$$
$$t + 16 = 14 \quad \text{Combine like terms}$$
$$t = -2 \quad \text{Subtract 16 from each member}$$

The solution set is $\{-2\}$. Check your answer by replacing t with -2 in the original equation.

Note A common error that is made when multiplying $-4(t-4)$ is to get $-4t - 16$. Do not forget that you are using the distributive property to multiply -4 times each term in the group $(t-4)$. The correct result is $-4t + 16$.

3. $\dfrac{5}{3a} + \dfrac{4}{9} = \dfrac{5}{12a}$

We determine that the LCD of the denominators is $36a$.

$$36a \cdot \frac{5}{3a} + 36a \cdot \frac{4}{9} = 36a \cdot \frac{5}{12a} \quad (a \neq 0) \quad \text{Multiply each term by the LCD } 36a$$
$$12 \cdot 5 + 4a \cdot 4 = 3 \cdot 5 \quad \text{Reduce each term}$$
$$60 + 16a = 15 \quad \text{Multiply as indicated}$$
$$16a = -45 \quad \text{Subtract 60 from each member}$$
$$a = -\frac{45}{16} \quad \text{Divide each member by 16}$$

The solution set is $\left\{-\dfrac{45}{16}\right\}$. Check your solution.

Note The domain of the variable is every real number *except* 0 since two of the terms are undefined when $a = 0$. This is an important observation to make as shown in example 4.

4. $\frac{3}{y} = \frac{4}{y^2 - 2y} - \frac{2}{y - 2}$

Factor the denominator $y^2 - 2y$ to get $y(y - 2)$. We determine the LCD of the denominators to be $y(y - 2)$.

$$y(y-2)\cdot\frac{3}{y} = y(y-2)\cdot\frac{4}{y(y-2)} - y(y-2)\cdot\frac{2}{y-2}\ (y \neq 0,2)$$

Multiply each term by the LCD $y(y - 2)$

$$(y-2)\cdot 3 = 4 - y\cdot 2$$

Reduce each term

$$3y - 6 = 4 - 2y$$

Multiply as indicated

$$3y + 2y = 4 + 6$$

Add 6 and $2y$ to each member

$$5y = 10$$

Combine like terms

$$y = 2$$

Divide each member by 5

Two *is not in the domain* of the variable y, since $y = 2$ makes the denominator $y - 2 = 0$. So 2 cannot be a solution of the equation. Therefore, the solution set is $\emptyset$.

We conclude there is no solution for the equation of example 4. The number 2 in that example is called an *extraneous solution.* An extraneous solution can occur whenever the variable appears in the denominator of one or more of the terms of the equation. Thus, you should *always* check your solution(s) of a rational equation when a variable appears in the denominator.

5. $\frac{1}{3}x^2 - \frac{5}{2}x + 3 = 0$

Multiply each term of the equation by 6, the LCD of the denominators 2 and 3.

$$6\cdot\frac{1}{3}x^2 - 6\cdot\frac{5}{2}x + 6\cdot 3 = 6\cdot 0$$

$$2x^2 - 15x + 18 = 0$$

Write the equation in standard form

$$(2x - 3)(x - 6) = 0$$

Factor $2x^2 - 15x + 18 = (2x - 3)(x - 6)$

$$2x - 3 = 0 \quad \text{or} \quad x - 6 = 0$$

Set each factor equal to 0

$$2x = 3 \qquad x = 6$$

Solve each equation

$$x = \frac{3}{2} \qquad x = 6$$

The solution set is $\left\{\frac{3}{2}, 6\right\}$. Check your solution.

Note We eliminate all the denominators *only* when solving rational equations. This is *never done* when adding or subtracting rational expressions. To illustrate, given the equation $\frac{6}{x} - \frac{4}{x^2} = 0$, we multiply each term by x^2, whereas given the subtraction problem $\frac{6}{x} - \frac{4}{x^2}$, we do *not* multiply each term by x^2.

▶ ***Quick check*** Find the solution set of $\frac{6}{4z} + \frac{7}{8} = \frac{9}{16z}$ ■

Rational equations in more than one variable

In scientific fields, equations and formulas involving rational expressions and *more than one variable* are common. It is often desirable to solve such equations for one variable in terms of the other variables in the equation. The procedures for finding such solutions are identical to those used in solving the preceding equations.

Solving a rational equation in more than one variable

1. Remove the fractions by multiplying each member of the equation by the LCD of the denominators in the equation.
2. Collect all terms containing the variable you are solving for in one member of the equation and all other terms in the other member.
3. Factor out the variable you are solving for if it appears in more than one term.
4. Divide each member by the coefficient of the variable for which you are solving.

■ ***Example 6–5 B***

Solve each rational equation for the indicated variable.

1. Solve $\frac{a}{3} + \frac{3x}{2} = c$ for x.

Determine that the LCD of 3 and 2 is 6.

$$
\begin{aligned}
6 \cdot \frac{a}{3} + 6 \cdot \frac{3x}{2} &= 6 \cdot c && \text{Multiply each term by 6}\\
2 \cdot a + 3 \cdot 3x &= 6c && \text{Reduce where possible}\\
2a + 9x &= 6c && \text{Multiply in left member}\\
2a + 9x - 2a &= 6c - 2a && \text{Subtract } 2a \text{ from each member}\\
9x &= 6c - 2a && \text{Combine like terms}\\
\frac{9x}{9} &= \frac{6c - 2a}{9} && \text{Divide each member by the coefficient 9}\\
x &= \frac{6c - 2a}{9}
\end{aligned}
$$

2. Solve $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$ for c $(a, b, c \neq 0)$.

Determine the LCD of a, b, and c is abc.

$$
\begin{aligned}
abc \cdot \frac{1}{a} &= abc \cdot \frac{1}{b} + abc \cdot \frac{1}{c} && \text{Multiply each term by } abc\\
bc &= ac + ab && \text{Reduce where possible}
\end{aligned}
$$

Note Since we are solving for c, we must get all terms containing c in the same member of the equation.

$$bc - ac = ac + ab - ac$$ Subtract ac from each member
$$bc - ac = ab$$
$$(b - a)c = ab$$ Factor c from each term in the left member
$$\frac{(b - a)c}{(b - a)} = \frac{ab}{(b - a)}$$ Divide each member by the coefficient of c, $(b - a)$
$$c = \frac{ab}{b - a}$$ Reduce in left member

▶ ***Quick check*** Solve $\frac{5}{a} - \frac{3}{b} = 3$ for a.

Mastery points

Can you

- Solve equations containing rational expressions?
- Solve rational equations for one variable in terms of the other variables?

Exercise 6–5

Find the solution set of each rational equation. Indicate any restrictions on the domain of the variable. See example 6–5 A.

Example $\frac{6}{4z} + \frac{7}{8} = \frac{9}{16z} (z \neq 0)$

Solution The LCD of the denominators is $16z$.

$$16z \cdot \frac{6}{4z} + 16z \cdot \frac{7}{8} = 16z \cdot \frac{9}{16z}$$ Multiply each term by $16z$
$$4 \cdot 6 + 2z \cdot 7 = 9$$ Reduce in each term
$$24 + 14z = 9$$ Multiply in each term
$$14z = -15$$ Subtract 24 from each member
$$z = -\frac{15}{14}$$ Divide each member by 14

The solution set is $\left\{-\frac{15}{14}\right\}$.

1. $\frac{y}{4} = \frac{2}{3}$
2. $\frac{4x}{5} - \frac{2}{3} = 4$
3. $\frac{p}{6} = \frac{7}{9}$
4. $\frac{a}{3} + \frac{5}{2} = 6$
5. $\frac{z}{8} + 3 = \frac{1}{4}$
6. $\frac{3R}{4} - 5 = \frac{5}{6}$
7. $\frac{3a}{6} + \frac{2a}{5} = 1$
8. $\frac{5x}{8} - \frac{x}{12} = 3$
9. $\frac{2x + 1}{7} - \frac{2x - 3}{14} = 1$
10. $\frac{3a + 1}{9} + \frac{1}{12} = \frac{2a - 1}{3}$
11. $\frac{3}{2x} = \frac{4}{5} + \frac{2}{x}$
12. $\frac{4}{b} - \frac{7}{3b} = \frac{2}{5}$

13. $\frac{2}{3R} + \frac{3}{2R} + \frac{1}{R} = 4$
14. $\frac{3}{w} - \frac{6}{5w} + \frac{1}{2w} = 5$
15. $\frac{4}{6y} + 5 = \frac{1}{9y} + 2$
16. $\frac{16}{5a} - 1 = 5 + \frac{3}{4a}$
17. $\frac{5}{3b} - \frac{1}{2} = \frac{7}{6b}$
18. $3 - \frac{5}{9x} = \frac{4}{6x}$
19. $\frac{3p + 2}{7p} - 3 = \frac{p}{14p}$
20. $\frac{5 - x}{8x} = \frac{2x + 5}{6x}$
21. $\frac{a - 4}{3a} = \frac{2a - 1}{4a}$
22. $\frac{R + 2}{10R} + \frac{4R - 1}{4R} = 2$
23. $\frac{4}{x - 4} = \frac{5}{x + 4}$
24. $\frac{9}{3 - x} = \frac{8}{2x + 1}$
25. $\frac{10}{3z + 1} = \frac{3}{5}$
26. $\frac{3}{2a + 3} = \frac{1}{7}$
27. $\frac{y}{y - 2} - 2 = \frac{2}{y - 2}$
28. $\frac{1}{x + 5} = \frac{3}{x - 5} - \frac{10}{x^2 - 25}$
29. $\frac{b - 1}{b^2 - 4} = \frac{6}{b + 2}$
30. $\frac{5}{R + 3} = \frac{4R + 3}{R^2 - 9}$
31. $\frac{5}{a^2 - 25} + \frac{3}{a - 5} = \frac{4}{a + 5}$
32. $\frac{6}{2p - 4} + \frac{4}{p - 2} = \frac{1}{p^2 - 4}$
33. $\frac{5}{x^2 + x - 6} = \frac{2}{x^2 + 3x - 10}$
34. $\frac{8}{a^2 - 6a + 8} = \frac{1}{a^2 - 16}$
35. $\frac{1}{a - 3} + \frac{2}{a + 4} = \frac{6}{a^2 + a - 12}$
36. $\frac{5}{2x - 1} - \frac{7}{3x + 2} = \frac{9}{6x^2 + x - 2}$
37. $3x^2 + 4x + \frac{4}{3} = 0$
38. $3x^2 + \frac{11}{2}x + \frac{3}{2} = 0$
39. $b^2 + \frac{3}{2}b = \frac{9}{2}$
40. $\frac{2}{3}x^2 + x = \frac{20}{3}$
41. $\frac{3}{4}z^2 = 2 - \frac{5}{2}z$
42. $x^2 - \frac{5}{6}x = \frac{2}{3}$
43. $x^2 = \frac{2}{3} - \frac{x}{3}$
44. $x^2 + \frac{4}{3}x = -\frac{4}{9}$

Solve the following equations for the indicated letter. Assume all denominators are nonzero. See example 6–5 B.

Example $\frac{5}{a} - \frac{3}{b} = 3$ for a

Solution

$ab \cdot \frac{5}{a} - ab \cdot \frac{3}{b} = ab \cdot 3$	Multiply each term by ab
$5b - 3a = 3ab$	Reduce each term
$5b = 3ab + 3a$	Add $3a$ to each member
So $5b = a(3b + 3)$	Factor a in the right member
$a = \frac{5b}{3b + 3}$	Divide each member by $3b + 3$

45. $\frac{2}{x} + \frac{1}{y} = 3$ for x
46. $\frac{5}{I} - 6 = \frac{8}{E}$ for I
47. $\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2}$ for c_1
48. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ for R
49. $\frac{1}{8} = \frac{1}{a} + \frac{1}{b}$ for a
50. $\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = 6$ for y
51. $\frac{3}{a} - \frac{4}{b} = \frac{5}{ab}$ for a
52. $\frac{5}{m} + 4 = \frac{6a}{2m} + 3b$ for m
53. $\frac{x + 4}{2} + \frac{y - 3}{5} = \frac{2}{10}$ for x

Solve the following word problems.

54. The principal amount of money P in a savings account paying interest rate r, over a given period of time t, which pays interest I is given by $P = \dfrac{I}{rt}$. Solve the equation for r.

55. The pressure per square inch of steam or water in a pipe, p, is given by $p = \dfrac{P}{LD}$, where
P = the total pressure on a diametral plane,
L = the length of the pipe in inches, and
D = the diameter of the pipe in inches.
Solve for D.

56. The safe internal unit pressure, p, in a given pipe of given thickness is given by $p = \dfrac{2st}{D}$, where
s = the unit tensile stress,
t = the thickness of the pipe, and
D = the diameter of the pipe. Solve for s.

57. The pitch diameter of a gear, P, is given by $P = \dfrac{D \cdot N}{N + 2}$, where D = the outside diameter of the gear and N = the number of teeth in the gear. Solve for N.

$\boxed{58.}$ The rule governing the speeds of two gears, one the driver gear and the other the driven gear, is given by $\dfrac{T_A}{T_B} = \dfrac{R_B}{R_A}$, where T_A = the number of teeth in the driven gear, T_B = the number of teeth in the driver gear, R_B = the revolutions per minute of the driven gear, and R_A = the revolutions per minute of the driver gear. Solve for T_B.

59. Given F is the force on the large piston and f is the force on the small piston of a hydraulic press, then $\dfrac{F}{f} = \dfrac{A}{a}$, where A is the area of the large piston and a is the area of the small piston. Solve for f.

60. The kinetic energy of a body, KE, is computed by $KE = \dfrac{Wv^2}{2g}$, where v is the velocity expressed in feet per second, W is the weight in pounds, and g is acceleration due to gravity. Solve for W.

61. Charles' Law is in regard to the relationship between pressure P, volume V, and the absolute temperature T of a gas as it expands when heated. Charles' Law is represented by $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$, where P_1, V_1, and T_1 are the initial conditions of the gas and P_2, V_2, and T_2 are the final conditions. Solve for T_2.

$\boxed{62.}$ The coefficient of linear expansion, k, of a solid when heated is given by $k = \dfrac{L_t - L_0}{L_0t}$, where L_0 is the length at 0° C, L_t is the length at t° C, and t is any given temperature in Celsius. Solve for L_t. Solve for L_0.

63. The coefficient of linear expansion α (alpha) when metal is heated is given by $\alpha = \dfrac{L_2 - L_1}{L_1(t_2 - t_1)}$. Solve for L_1.

64. Carnot's ideal efficiency of any heat engine operating between the temperature limits T_1 and T_2 is given by $CE = \dfrac{T_1 - T_2}{T_1}$. Solve for T_1.

65. A formula for resistors in parallel is given by $R = \dfrac{R_1R_2}{R_1 + R_2}$. Solve for R_1.

66. In electronics, a formula for the unknown resistance R_x in a battery is given by $R_x = R_m\left(\dfrac{E_1}{E_2} - 1\right)$. Solve for E_2.

Review exercises

Evaluate each expression for the given values. See sections 2–2 and 5–1.

1. $5x - 3y + z$ when $x = 1$, $y = -2$, and $z = 3$

2. $\dfrac{3a - b}{2a + b}$ when $a = 4$ and $b = -5$

3. $\dfrac{y_1 - y_2}{x_1 - x_2}$ when $x_1 = 3$, $x_2 = 1$, $y_1 = 3$, and $y_2 = -5$

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Solve each equation for y. See section 2–7.

4. $5x + y = 4$ **5.** $2x + 3y = 6$ **6.** $x - 4y = 8$

Simplify the following expressions. Assume all denominators are nonzero. Answer with positive exponents only. See section 3–4.

7. $(3y^{-1})(y^2x^{-2})$ **8.** $\dfrac{x^{-3}y^3}{x^2y^{-1}}$

6–6 ■ Rational expression applications

Rational expressions occurring in rational equations have many applications in the physical and scientific world. We now wish to discuss some of the more common applications.

■ Example 6–6 A

Choose a variable, set up an appropriate equation, and solve the following problems.

1. A water holding tank is fed by two pipes. If it takes the smaller pipe 12 hours to fill the tank and the larger pipe 9 hours to fill the tank, how long would it take to fill the tank if both pipes are open? (This is called a *work* problem.)

Let $x =$ the number of hours it takes the two pipes to fill the tank. Now,

a. the smaller pipe can fill $\dfrac{1}{12}$ of the tank in 1 hour,

b. the larger pipe can fill $\dfrac{1}{9}$ of the tank in 1 hour,

c. the two pipes together can fill $\dfrac{1}{x}$ of the tank in 1 hour.

The amount of the tank filled by the smaller pipe in 1 hour plus the amount of the tank filled by the larger pipe in 1 hour must be equal to the amount of the tank filled by the two pipes together in 1 hour. Thus, the equation is

$$\underset{\text{Amount by smaller pipe}}{\dfrac{1}{12}} + \overset{\text{Amount by larger pipe}}{\dfrac{1}{9}} = \underset{\text{Amount together}}{\dfrac{1}{x}}$$

Multiply each term by the LCM of 12, 9, and x, which is $36x$.

$$36x \cdot \frac{1}{12} + 36x \cdot \frac{1}{9} = 36x \cdot \frac{1}{x}$$

$$3x + 4x = 36 \quad \text{Reduce in each term}$$

$$7x = 36 \quad \text{Combine in left member}$$

$$x = \frac{36}{7} \quad \text{Divide each member by 7}$$

Therefore, together the two pipes can fill the tank in $\dfrac{36}{7}$ hours, or $5\dfrac{1}{7}$ hours.

Note That is, approximately 5 hours and 9 minutes.

2. Jim James drove a distance of 120 miles, part at 50 miles per hour (mph) and part at 60 mph. If he drove the 120 miles in $2\frac{1}{4}$ hours, how many miles did he drive at 50 mph? (This is called a *time–rate–distance* problem.)

Note In a distance (d)–rate (r)–time (t) problem, we use $d = rt$, $t = \frac{d}{r}$, or $r = \frac{d}{t}$.

Let x = the distance he drove at 50 mph. Then $120 - x$ = the distance he drove at 60 mph.

The time traveled at 50 mph plus the time traveled at 60 mph equals the total time traveled, $2\frac{1}{4}$ hr $\left(\text{or } \frac{9}{4} \text{ hr}\right)$. We use the following table for time–rate–distance problems.

distance (d)	rate (r)	time (t)
x	50	$\frac{x}{50}$
$120 - x$	60	$\frac{120 - x}{60}$

$t = \frac{d}{r}$

The equation is then

$$\underset{\text{Time at 50 mph}}{\frac{x}{50}} + \overset{\text{Time at 60 mph}}{\frac{120 - x}{60}} = \underset{\text{Total time}}{\frac{9}{4}}$$

$$300 \cdot \frac{x}{50} + 300 \cdot \frac{120 - x}{60} = 300 \cdot \frac{9}{4} \qquad \text{Multiply each term by the LCD 300}$$

$$6x + 5(120 - x) = 675 \qquad \text{Reduce in each term}$$

$$6x + 600 - 5x = 675 \qquad \text{Distributive property}$$

$$x + 600 = 675 \qquad \text{Combine in left member}$$

$$x = 75 \qquad \text{Subtract 600 from each member}$$

Thus, Jim James drove 75 miles at 50 mph.

Note He drove $120 - x = 120 - 75 = 45$ miles at 60 mph.

3. The denominator of a fraction is 3 more than the numerator. If 4 is added to the numerator and the denominator, the resulting fraction is $\frac{3}{4}$. Find the original fraction.

Let x = the numerator of the original fraction. Then $x + 3$ = the denominator of the original fraction. The equation we get is

$$\frac{x + 4}{(x + 3) + 4} = \frac{3}{4}$$

then

$$\frac{x+4}{x+7} = \frac{3}{4}$$

The LCD of 4 and $x + 7$ is $4(x + 7)$.

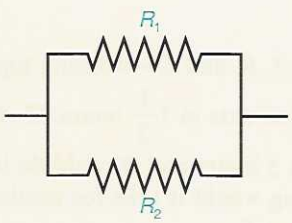
$$4(x+7)\cdot\frac{x+4}{x+7} = 4(x+7)\cdot\frac{3}{4}\ (x \neq -7) \qquad \text{Multiply each term by } 4(x+7)$$

$$\begin{aligned} 4(x+4) &= (x+7)\cdot 3 && \text{Reduce each term} \\ 4x+16 &= 3x+21 && \text{Multiply as indicated} \\ x &= 5 && \text{Subtract } 3x \text{ and 16 from each member} \\ \text{and } x+3 &= 8 \end{aligned}$$

The original fraction is $\frac{5}{8}$.

4. In an electrical circuit, when two resistors are connected in parallel, the total resistance R of the circuit in ohms is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$



where R_1 and R_2 are the resistances of the two resistors in ohms and the circuit is connected in parallel as shown in the diagram. Find the total resistance R of an electrical circuit having two resistors connected in parallel if their resistances are 4 ohms and 6 ohms.

We want R if $R_1 = 4$ ohms and $R_2 = 6$ ohms.

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{6}$$

Multiply both members by the LCM of 4, 6, and R, which is $12R$.

$$\begin{aligned} 12R\cdot\frac{1}{R} &= 12R\cdot\frac{1}{4} + 12R\cdot\frac{1}{6}\ (R \neq 0) \\ 12 &= 3R + 2R \\ 12 &= 5R \\ \frac{12}{5} &= R \end{aligned}$$

Therefore the total resistance is $\frac{12}{5}$, or $2\frac{2}{5}$, ohms.

▶ ***Quick check*** If the same number is added to the numerator and the denominator of $\frac{3}{4}$, the result is the fraction $\frac{5}{6}$. What is the number? ■

Mastery points

Can you

- Solve work problems?
- Solve distance-rate-time problems?
- Solve number problems?
- Solve resistance in electric circuits problems?

Exercise 6–6

Choose a variable, set up an equation, and solve the following problems.

Example If the same number is added to the numerator and the denominator of $\frac{3}{4}$, the result is the fraction $\frac{5}{6}$. What is the number?

Solution Let x = the number to be added to the numerator and the denominator. The equation is then

$$\frac{3+x}{4+x} = \frac{5}{6}$$

$$6(4+x)\,\frac{3+x}{4+x} = 6(4+x)\cdot\frac{5}{6} \qquad \text{Multiply each member by the LCD } 6(4+x)$$

$$6(3+x) = (4+x)\cdot 5 \qquad \text{Reduce in each member}$$

$$18 + 6x = 20 + 5x \qquad \text{Multiply as indicated}$$

$$x = 2 \qquad \text{Solve for } x$$

The number to be added is 2. $\left(\textit{Check: } \frac{3+2}{4+2} = \frac{5}{6}\right)$

See example 6–6 A–1.

1. Jim can mow his parents' lawn in 50 minutes. His younger brother Kenny can mow the lawn in 70 minutes. How long would it take for the boys to mow the lawn if they mow together?
2. In a factory, worker A can do a certain job in 6 hours while worker B can do the same job in 5 hours. How long would it take workers A and B to do the same job working together?
3. Three different sized pipes feed water into a swimming pool. If the pipes can fill the same pool individually in 6 hours, 8 hours, and 9 hours, respectively, how long would it take to fill the same pool if all three pipes were open?
4. During "clean-up week" in Podunk Junction, Jane, Joan, and Ruth work to clean up a vacant lot. If the girls can do the job individually in 2 hours, 3 hours, and 4 hours, respectively, how long will it take them, working together, to clean the lot?
5. It takes two men—Harry and Dick—working together 4 hours to paint the exterior of a house. If Harry could do the job in 6 hours working alone, how long would it take Dick to paint the house alone?
6. A water tank has two drain pipes. If the larger pipe could empty the tank in 45 minutes and the two pipes together could drain the tank in 30 minutes, how long would it take for the smaller pipe to drain the tank?
7. Three combines—A, B, and C—working together can harvest a field of oats in $1\frac{1}{2}$ hours. If A could do the job alone in 5 hours and B could do it alone in 6 hours, how long would it take for combine C to do the same job alone?
8. In a pizzeria, three women are at work making pizzas. The three could make 50 pizzas in $1\frac{1}{2}$ hours working together. If two of the women could make all the pizzas in 5 hours and 6 hours, respectively, working alone, how long would it take the third woman to make the 50 pizzas working alone?
9. A tank has one inlet pipe and one outlet pipe. If the inlet pipe can fill the tank in $3\frac{1}{3}$ hours and the outlet pipe can empty the tank in 5 hours, how long would it take to fill the tank if both pipes are left open? (*Hint:* Subtract the drainage.)
10. A sink drain, when left open, can empty a sink full of water in 4 minutes. If the cold water and hot water faucets can, when fully open, fill the sink in $2\frac{1}{2}$ minutes and 3 minutes, respectively, how long would it take to fill the sink if all three are open simultaneously?

See example 6–6 A–2.

11. Jack drove 320 miles in $5\frac{1}{2}$ hours. If he drove part of the trip averaging 55 mph and the rest averaging 60 mph, how many miles did he drive at each speed?

12. A. J. Foyt, when driving the Indianapolis 500 Race, averaged 198 mph over part of the race. Due to an accident on the track, he averaged 160 mph over the rest. If the race took 2 hours and 40 minutes to run, how many miles did he drive at an average of 198 mph? (Round the answer to the nearest integer.)

13. Car *A* travels 120 miles in the same time that car *B* travels 150 miles. If car *B* averages 10 mph faster than car *A,* how fast is each car traveling?

14. A freight train travels 260 kilometers in the same time a passenger train travels 320 kilometers. If the passenger train averages 15 kilometers per hour faster than the freight train, what was the average speed of the freight train?

15. Sheila can row a boat 2 mph in still water. How fast is the current of a river if she takes the same length of time to row 4 miles upstream as she does to row 10 miles downstream? (*Hint:* Subtract current upstream and add downstream.)

16. An airplane flew 1,000 miles with the wind in the same length of time it took to fly 850 miles against the wind. If the wind was blowing at 25 miles per hour, what was the average air speed of the plane?

17. On a trip from Detroit to Columbus, Ohio, Mrs. Smith drove at an average speed of 60 mph. Returning, her average speed was 55 mph. If it took her $\frac{1}{3}$ hour longer on the return trip, how far is it from Detroit to Columbus?

18. A jet plane flew at an average speed of 240 miles per hour going from city *A* to city *B* and averaged 300 miles per hour on the return flight. Its return flight took 1 hour and 40 minutes less time. How far is it from city *A* to city *B?* (Disregard any wind.)

See example 6–6 A–3.

19. The numerator of a given fraction is 4 less than the denominator. If 5 is added to both the numerator and the denominator, the resulting fraction is $\frac{5}{7}$. What is the original fraction?

20. The denominator of a fraction exceeds the numerator by 7. If 3 is added to the numerator and 1 is subtracted from the denominator, the resulting fraction is $\frac{4}{5}$. Find the original fraction.

21. One number is four times another number. The sum of their reciprocals is $\frac{5}{12}$. What are the numbers?

22. One number is four times another number. The sum of their reciprocals is $\frac{1}{4}$. What are the numbers?

23. If $\frac{1}{2}$ is added to three times the reciprocal of a number, the result is 1. Find the number.

24. If $\frac{1}{2}$ is subtracted from four times the reciprocal of a number, the result is 0. Find the number.

25. When a certain number is added to the numerator and subtracted from the denominator of the fraction $\frac{2}{5}$, the result is 6. What is the number?

26. When a certain number is added to the numerator and subtracted from the denominator of the fraction $\frac{5}{9}$, the result is $\frac{4}{7}$. What is the number?

27. A prescription for an illness calls for a child's dosage to be $\frac{3}{5}$ of the number of pills that an adult takes. Together they use 24 pills. How many pills are taken by each?

28. An apprentice electrician receives $\frac{3}{8}$ of the hourly wage of a journeyman electrician. Together their hourly wage is \$29.70. Find the hourly wage of each.

29. Pat and Mike earn a total of \$65.50 per week delivering papers. If Pat's earnings are $\frac{5}{7}$ of Mike's, how much does Mike earn each day?

Use the formula $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ to solve the following exercises. See example 6–6 A–5.

30. Two resistors in an electric circuit have resistances of 6 ohms and 8 ohms and are connected in parallel. Find the total resistance of the circuit.

31. Two resistors of an electric circuit are connected in parallel. If one has a resistance of 5 ohms and the other has a resistance of 12 ohms, what is the total resistance in the circuit?

32. Three resistors connected in parallel have resistances of 4 ohms, 6 ohms, and 10 ohms. What is the total resistance in the electric circuit? $\left(\textit{Hint:} \text{ Use } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.\right)$

33. The total resistance in a parallel wiring circuit is 12 ohms. If the resistance in one branch is 30 ohms, what is the resistance in the other branch?

34. The resistance in one branch of a two-resistor parallel wiring circuit is 10 ohms. If the total resistance in the circuit is 6 ohms, what is the resistance in the other branch?

35. A three-resistor parallel wiring circuit has a total resistance of 10 ohms. If two of the branches of the circuit have resistances of 20 ohms and 30 ohms, what is the resistance in the third branch? (See hint in exercise 32.)

Review exercises

Find the solution set of the following equations. See sections 2–6 and 2–7.

1. $2y + 3 = 4y - 1$

2. $2y - 3x = 6$ for y

Factor the following expressions. See sections 4–2, 4–3, and 4–4.

3. $8y^2 - 32$

4. $x^2 + 20x + 100$

5. $3y^2 - 4y - 4$

Combine the following rational expressions. See section 6–3.

6. $\frac{3x}{x-1} + \frac{2x}{x+3}$

7. $\frac{4y}{2y+1} - \frac{3y}{y-5}$

Chapter 6 lead-in problem

Marc owns $\frac{5}{8}$ interest in a print shop and his uncle owns $\frac{1}{4}$ interest in the shop. In a given year, they shared earnings of \$140,000. How much did the shop earn that year?

Solution

Let x = the total earnings of the print shop.

	Interest	Shop earnings	Income
Marc	$\frac{5}{8}$	x	$\frac{5}{8}x$
Uncle	$\frac{1}{4}$	x	$\frac{1}{4}x$

Then $\frac{5}{8}x + \frac{1}{4}x = 140{,}000$ — Together they earned \$140,000

$8 \cdot \frac{5}{8}x + 8 \cdot \frac{1}{4}x = 8 \cdot 140{,}000$ — Multiply each term by the LCM of 4 and 8, which is 8

$5x + 2x = 1{,}120{,}000$ — Perform indicated multiplications

$7x = 1{,}120{,}000$ — Combine like terms

$x = 160{,}000$ — Divide each member by 7

Chapter 6 summary

1. To *multiply* two rational expressions, we multiply the numerators and place that product over the product of the denominators.
2. To *divide* two rational expressions, $\frac{P}{Q} \div \frac{R}{S}$, we multiply $\frac{P}{Q}$ by the reciprocal of $\frac{R}{S}$, which is $\frac{S}{R}$.
3. To find the *least common denominator* (LCD) of two or more rational expressions, we
   a. write each denominator in completely factored form;
   b. take each different factor that appears in the factorizations in step a and write them as a product;
   c. raise each factor of step b to the greatest power it has in step a.
4. To obtain an *equivalent rational expression,* we multiply, or divide, the numerator and the denominator of the given rational expression by a *common factor*.
5. To *add* or *subtract* rational expressions, we must
   a. write each rational expression as an equivalent rational expression with the same denominator, preferably the LCD;
   b. add, or subtract, the numerators and place this sum, or difference, over the common denominator.
6. A **complex fraction** is a fraction whose numerator or denominator, or both, contains at least one fraction.
7. In the *complex fraction*

$$\frac{\frac{a}{b}}{\frac{c}{d}}$$

   we call $\frac{a}{b}$ the *primary numerator,* $\frac{c}{d}$ the *primary denominator,* and b and d the *secondary denominators.*
8. To *simplify a complex fraction,* we
   a. divide the primary numerator by the primary denominator after making a single fraction of each; or
   b. multiply the primary numerator and the primary denominator by the LCM of the secondary denominators and reduce the result, where possible.
9. A **rational equation** is an equation that contains at least one rational expression.
10. To *solve a rational equation,* we
    a. multiply each term of the equation by the LCD to eliminate the fractions;
    b. use the basic procedures for solving equations;
    c. check for extraneous solutions.

Chapter 6 error analysis

1. Subtracting rational expressions
   Example: $\frac{3x-1}{x-2} - \frac{x+2}{x-2} = \frac{3x-1-x+2}{x-2}$
   $= \frac{2x+1}{x-2}$
   Correct answer: $\frac{2x-3}{x-2}$
   What error was made? (*see page 240*)
2. Solving rational equations
   Example: Find the solution set of $\frac{y}{3} - \frac{y+1}{4} = \frac{2}{12}$
   $12 \cdot \frac{y}{3} - 12 \cdot \frac{y+1}{4} = 12 \cdot \frac{2}{12}$
   $4y - 3(y+1) = 2$
   $4y - 3y + 3 = 2$
   $y = -1 \qquad \{-1\}$
   Correct answer: $\{5\}$
   What error was made? (*see page 259*)
3. Adding and subtracting rational expressions
   Example: $\frac{3}{x-5} + \frac{5}{5-x} = \frac{3+5}{x-5} = \frac{8}{x-5}$
   Correct answer: $\frac{-2}{x-5}$
   What error was made? (*see page 241*)
4. Adding and subtracting rational expressions
   Example: $\frac{3}{x^2} + \frac{2}{x^2} = x^2 \cdot \frac{3}{x^2} + x^2 \cdot \frac{2}{x^2} = 3 + 2 = 5$
   Correct answer: $\frac{5}{x^2}$
   What error was made? (*see page 239*)
5. Exponents
   Example: $(-4)^2 = -16$
   Correct answer: 16
   What error was made? (*see page 57*)
6. Order of operations
   Example: $3^2 + 12 \cdot 2 - 8 \div 2 = 17$
   Correct answer: 29
   What error was made? (*see page 57*)
7. Combining polynomials
   Example: $(4a^2b - 2ab^2) - (a^2b + ab^2) = 3a^2b - ab^2$
   Correct answer: $3a^2b - 3ab^2$
   What error was made? (*see page 82*)
8. Solving linear inequalities
   Example: If $4 < -2x \leq 6$, then $-2 < x \leq -3$.
   Correct answer: If $4 < -2x \leq 6$, then $-2 > x \geq -3$.
   What error was made? (*see page 116*)

9. Multiplication of like bases
Example: $5 \cdot 5^3 = 5^3$
Correct answer: 5^4
What error was made? (*see page 129*)

10. Squaring a binomial
Example: $(x - 6)^2 = (x)^2 - (6)^2 = x^2 - 36$
Correct answer: $x^2 - 12x + 36$
What error was made? (*see page 135*)

Chapter 6 critical thinking

Which integers can be written as the sum of three consecutive integers?

Example: $6 = 1 + 2 + 3$

Chapter 6 review

[6–1]

Perform the indicated multiplication and reduce the product to lowest terms.

1. $\dfrac{15}{8a} \cdot \dfrac{12a}{5}$
2. $\dfrac{24b}{7a} \cdot \dfrac{21a^2}{8b^2}$
3. $\dfrac{x - 3y}{2x + y} \cdot \dfrac{2x - y}{x - 3y}$
4. $\dfrac{5x - 10}{x + 3} \cdot \dfrac{3x + 9}{15}$
5. $\dfrac{m^2 - n^2}{14x} \cdot \dfrac{35x^2}{5m + 5n}$
6. $\dfrac{y}{y^2 - 1} \cdot \dfrac{y + 1}{y^2 - y}$
7. $\dfrac{b^2 - 36}{b^2 + 12b + 36} \cdot \dfrac{2b + 12}{b - 6}$
8. $\dfrac{5 - x}{6a - 3} \cdot \dfrac{2a - 1}{x^2 - 25}$
9. $\dfrac{x^2 + x - 2}{x^2 - 4x - 12} \cdot \dfrac{x^2 - 7x + 6}{x^2 - 1}$

Find the indicated quotients and state the answer reduced to lowest terms. Assume all denominators are nonzero.

10. $\dfrac{14a}{9} \div \dfrac{7}{3}$
11. $\dfrac{10}{9b} \div \dfrac{35}{12b^2}$
12. $\dfrac{24ab}{7} \div \dfrac{16a^2b^2}{21}$
13. $\dfrac{2x - 1}{3x + 4} \div \dfrac{3x}{7x}$
14. $\dfrac{x + 6}{x^2 - 4} \div \dfrac{(x + 6)^2}{x + 2}$
15. $\dfrac{4a - 8}{2a + 1} \div \dfrac{6a - 12}{10a + 5}$
16. $\dfrac{x^2 + 16x + 64}{x^2 + 9x + 8} \div \dfrac{x^2 - 64}{x + 1}$
17. $\dfrac{1 - b}{(b + 3)^2} \div \dfrac{b^2 - 2b + 1}{b^2 - 9}$
18. $\dfrac{9a^2 + 15a + 6}{a^2 + 3a - 4} \div \dfrac{36a^2 - 16}{3a^2 + 10a - 8}$

[6–2]

Find the least common denominator of rational expressions having the given denominators. Assume all denominators are nonzero.

19. $9x^2$ and $12x$
20. $14ab$ and $21a^2b^2$
21. $x^2 - 9$ and $2x + 6$
22. $y^2 - 2y - 15$ and $y^2 - 25$
23. $4z - 8$, $z^2 - 4$, and $z + 1$
24. $x^2 + x$, $x^2 + 2x + 1$, and $3x^2 - 2x - 5$

Find the indicated sum or difference. Assume all denominators are nonzero.

25. $\dfrac{4}{x} + \dfrac{1}{-x}$
26. $\dfrac{y}{y - 2} - \dfrac{3y}{2 - y}$
27. $\dfrac{3x - 2}{3x - 1} - \dfrac{x + 6}{1 - 3x}$
28. $\dfrac{x + 7}{2x - 5} + \dfrac{2x - 6}{5 - 2x}$

[6–3]

Add or subtract as indicated and reduce the answer to lowest terms. Assume all denominators are nonzero.

29. $\dfrac{4x}{15} + \dfrac{8x}{21}$
30. $\dfrac{25}{16a} - \dfrac{13}{12a}$
31. $\dfrac{5}{3x + 1} + \dfrac{9}{4x - 3}$
32. $\dfrac{4x}{x + 4} - \dfrac{7x}{x^2 - 16}$
33. $\dfrac{4}{ab^2} + \dfrac{12}{5a^2b} - \dfrac{3}{4ab}$
34. $4a + \dfrac{6}{a + 1}$

35. $\dfrac{7}{x^2+1} - 10$

36. $\dfrac{x+1}{4x} - \dfrac{3x-5}{8x^2} + \dfrac{5x+4}{12x}$

37. $\dfrac{4y}{y^2-7y-18} + \dfrac{9y}{y^2-4}$

38. $\dfrac{5a}{x+3} + \dfrac{2}{x^2-9} - \dfrac{7a}{x-3}$

39. $\dfrac{9}{x} - \dfrac{4}{x-5} + \dfrac{3}{x+4}$

40. $\dfrac{5}{x^2+2xy+y^2} + \dfrac{5}{x^2-xy-2y^2}$

[6–4]

Simplify the given complex fractions and reduce to lowest terms.

41. $\dfrac{\frac{4}{7}}{\frac{9}{4}}$

42. $\dfrac{\frac{3}{4}-\frac{1}{2}}{\frac{1}{4}+\frac{3}{2}}$

43. $\dfrac{\frac{4}{5}+1}{2-\frac{1}{5}}$

44. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}$

45. $\dfrac{\frac{4}{x^2}+\frac{3}{x}}{\frac{2}{x^2}-\frac{5}{x}}$

46. $\dfrac{\frac{a}{b}-\frac{b}{a}}{\frac{a}{b}+\frac{b}{a}}$

47. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{xy}}$

48. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$

49. $\dfrac{\frac{a^2}{b^2}-2a-3}{\frac{a-3b}{ab}}$

[6–5]

Find the solution set of the following rational equations.

50. $\dfrac{x}{8} - 3 = \dfrac{2x}{12} + 1$

51. $\dfrac{4a+1}{4} + \dfrac{5}{6} = 3a - \dfrac{5}{8}$

52. $\dfrac{12}{4a} - \dfrac{5}{6a} = 4$

53. $\dfrac{7}{a+1} - \dfrac{1}{a^2-1} = \dfrac{2}{a+1}$

54. $\dfrac{1}{y+3} - \dfrac{6}{y} = \dfrac{7}{y} + \dfrac{2}{y+3}$

55. $\dfrac{7}{2c^2} + \dfrac{5}{6c^2} = \dfrac{7}{12c^3}$

56. $\dfrac{x^2}{4} - 3x = 0$

57. $\dfrac{2}{3}y^2 = \dfrac{3}{2}$

58. $\dfrac{x^2}{2} - \dfrac{15}{2} = -x$

59. $\dfrac{a^2}{3} = \dfrac{5a}{2} - 3$

Solve for the indicated variable.

60. $\dfrac{a}{x} + \dfrac{b}{x} = 3$ for x

61. $\dfrac{y}{x+1} = \dfrac{y^2}{x-3}$ for x

62. $\dfrac{3}{4-y} = \dfrac{a}{b}$ for y

63. $\dfrac{4}{y-c} - \dfrac{5}{y+b} = 0$ for y

Solve the following word problems.

64. An equation for tensile and compressional stresses is given by $y = \dfrac{Fl}{Ae}$. Solve for l.

65. The efficiency of a screw jack is calculated by the formula $E = \dfrac{Wp}{2+LF}$. Solve for L.

66. A formula for the theoretical mechanical advantage, M, of a chain fall is given by $M = \dfrac{2R}{R-r}$. Solve for R.

67. The total reaction force F of air against a plane at the bottom of a vertical loop in centripetal force is given by $F = \dfrac{mv^2}{r} + m \cdot g$. Solve for m.

[6–6]

68. With different equipment, one painter can paint a house three times faster than a second painter. Working together they can do it in 4 hours. How long would it take each of them to paint the house working alone?

69. Paul can row his boat at a rate of 4 miles per hour in still water. It takes him as long to row 20 miles downstream as it takes him to row 8 miles upstream. What is the rate of the current?

70. The sum of three times a number and twice its reciprocal is 5. Find the number.

Chapter 6 cumulative test

Compute and simplify.

[1–5] 1. $-\frac{1}{3}+\frac{1}{4}+(-6)+\frac{1}{6}$

[1–6] 2. $-\frac{4}{5}\left(-\frac{15}{16}\right)$

[1–5] 3. $\frac{5}{6}-\left(-\frac{3}{5}\right)$

[1–7] 4. $-\frac{24}{35}\div\left(-\frac{3}{7}\right)$

Simplify the following expressions with only positive exponents.

[3–3] 5. $x^8\cdot x^3\cdot x^0$

[3–3] 6. $\frac{y^{-5}}{y^4}$

[3–4] 7. $(-5x^2y^{-3})^2$

[2–3] 8. $(9x^3-3x^2+4x-8)-(-x^3+x^2-7x+1)$

Find the solution set of the following equations and inequalities.

[2–6] 9. $-5(2x+5)=-3x+1$

[2–6] 10. $\frac{1}{3}x-\frac{3}{4}=6$

[4–7] 11. $2y^2-y=6$

[2–9] 12. $3x-4\le 5x+6$

Completely factor each expression.

[4–4] 13. $2x^2-18$

[4–1] 14. $6x^5-36x^3+9x^2$

[4–3] 15. $4x^2+16x+15$

[4–4] 16. $3-12x^6$

Perform the indicated operations.

[3–2] 17. $(7x-6)^2$

[3–2] 18. $\left(5-\frac{1}{2}x\right)\left(5+\frac{1}{2}x\right)$

[3–2] 19. $(2x-3)(3x^2-5x+11)$

[3–2] 20. $(3-2x^2)(5-4x^2)$

[5–4] 21. Find x when $\frac{x}{24}=\frac{5}{7}$.

[5–4] 22. Power (in foot-pounds per minute) is given by the ratio of work, w, to time, t. Find the power exerted by pushing an 80-pound load 180 feet in 6 minutes. (*Hint:* w = weight · length.)

Perform the indicated operations and reduce the answers to lowest terms.

[6–1] 23. $\frac{4a}{5b}\cdot\frac{a}{b}$

[6–1] 24. $\frac{y+3}{y-6}\cdot\frac{y+7}{y-2}$

[6–1] 25. $\frac{y^2-4}{y+3}\cdot\frac{y^2-9}{y+2}$

[6–1] 26. $\frac{x^2-y^2}{9}\div\frac{y^2+xy}{9x-9}$

[6–3] 27. $\frac{2}{x^2y}+\frac{4}{xy^2}$

[6–3] 28. $\frac{6}{x-y}-\frac{3}{x+y}$

[6–3] 29. $\frac{a+1}{a-2}+\frac{a-8}{2-a}$

[6–3] 30. $\frac{7}{x^2-49}+\frac{6}{x^2-5x-14}$

[6–3] 31. $\frac{2x}{x^2+x-6}-\frac{3x}{x^2-3x-18}$

Simplify the following complex rational expressions.

[6–4] 32. $\dfrac{3 - \dfrac{1}{y}}{4 + \dfrac{5}{y}}$

[6–4] 33. $\dfrac{\dfrac{1}{x} + \dfrac{1}{y}}{\dfrac{3}{y} - \dfrac{4}{x}}$

[6–4] 34. $\dfrac{\dfrac{x - 4}{x + 7}}{\dfrac{x + 5}{x^2 - 49}}$

Find the solution set of the following equations.

[6–5] 35. $\dfrac{4}{x} = \dfrac{6}{2x - 1}$

[6–5] 36. $x^2 - 2 = \dfrac{17x}{3}$

[6–5] 37. Solve the equation $\dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{q}$ for q.

Choose a variable, set an appropriate equation, and solve the following problems.

[6–6] 38. John can build a fence in 6 days and Harry can build the same fence in 4 days. How long would it take them to build the fence working together?

[6–6] 39. The denominator of a fraction is two times the numerator. If 2 is added to the numerator and 2 is subtracted from the denominator, the resulting fraction is $\dfrac{5}{7}$. Find the original fraction.

15. $\frac{R-4}{3R-1}$ **16.** $\frac{5-n}{2+n}$ **17.** $-8x^2$ **18.** $2a - 3 + 5a^2$
19. $5x - 3y^3 + xy$ **20.** $2a + 3b - 6ab^6$
21. $4a + 1 + \frac{-2}{2a-1}$ **22.** $3a - 2 + \frac{1}{a-5}$ **23.** $x - 7$
24. $5x^2 - x - 4$ **25.** $\frac{3}{7}$ or 3:7 **26.** $\frac{9}{4}$ or 9:4 **27.** $\frac{2}{5}$ or 2:5
28. $30 \frac{\text{miles}}{\text{gallon}}$ or 30 miles per gallon **29.** $\frac{2{,}193}{881}$ or 2,193:881
30. $\frac{7}{3}$ or 7:3 **31.** $x = 32$ **32.** $a = 3.6$ **33.** $y = \frac{54}{5}$
34. $p = \frac{2}{5}$ **35.** 35 feet **36.** 12 quarts antifreeze, 4 quarts water

Chapter 5 cumulative test

1. 1,756 **2.** undefined **3.** -36 **4.** $5a + 3b$
5. $9x^2 - 12x + 4$ **6.** $25y^2 - 4$ **7.** $4a^2 - 21ab - 18b^2$
8. $x^3 - 3x^2y + 3xy^2 - y^3$ **9.** -33 **10.** $\{-2\}$ **11.** $\{8\}$
12. $-\frac{9}{2} \le x < 3$ **13.** $x > 3$ **14.** $(m + n)(x - y)$
15. $(3a + 4)(a + 1)$ **16.** $(2x - 5)^2$ **17.** $(2z + 3)(2z - 3)$
18. $(6 - y)(6 + y)$ **19.** $(x - 15)(x + 3)$
20. $2(a - 2b)(a^2 + 2ab + 4b^2)$ **21.** $\{7, -2\}$ **22.** $\left\{\frac{3}{2}, -3\right\}$
23. $\frac{z^3}{64y^3}$ **24.** a^3 **25.** $8a^3$ **26.** $x = 24$ **27.** $\frac{13}{6}$ or 13:6
28. $x - 6 + \frac{1}{x-2}$ **29.** $\frac{4}{3}$ **30.** $\frac{9b}{7a^2}$ **31.** $\frac{a-6}{a-7}$
32. $\frac{8}{5(x+y)}$ **33.** $\frac{y+4}{y+5}$ **34.** $\frac{x-3y}{3x+2y}$ **35.** $21\frac{1}{3}$ inches
36. $12\frac{6}{7}$ feet

Chapter 6

Exercise 6–1

Answers to odd-numbered problems

1. $\frac{3}{5}$ **3.** $\frac{5}{6}$ **5.** $2a$ **7.** 10 **9.** $\frac{x}{4y}$ **11.** $\frac{3x}{4}$ **13.** $\frac{4}{35x}$
15. $\frac{35x}{4}$ **17.** $\frac{7c}{2b}$ **19.** $\frac{x}{6y^2}$ **21.** $\frac{16bcx}{3az}$ **23.** $\frac{2a}{5b^2x}$
25. $\frac{4}{x+y}$ **27.** $-\frac{3}{4}$ **29.** $-\frac{15}{4}$ **31.** $\frac{4}{a-5}$ **33.** $\frac{18(x-2)}{x+2}$
35. $\frac{24y(x-2)}{25}$ **37.** $(r - 4)(r - 1)$ **39.** $-4(x + 3)$
41. $\frac{a-3}{a-5}$ **43.** $\frac{(x-3)(x+1)}{(x-1)(x+2)}$ **45.** $\frac{(2x-1)(x-1)}{(x-8)(x+7)}$
47. 1 **49.** $3x + 4$ **51.** $\frac{1}{(2x+1)^2}$ **53.** $\frac{2a+12}{3a^2+9a+27}$
55. $\frac{(z-7)(z+3)}{5(z^2-2z+4)}$ **57.** $y + 5$

Solutions to trial exercise problems

10. $\frac{7a}{12b} \cdot \frac{9b}{28} = \frac{7a \cdot 9b}{12b \cdot 28} = \frac{7 \cdot a \cdot 3 \cdot 3 \cdot b}{2 \cdot 2 \cdot 3 \cdot b \cdot 2 \cdot 2 \cdot 7}$
$= \frac{3 \cdot a \cdot (7 \cdot 3 \cdot b)}{2 \cdot 2 \cdot 2 \cdot 2 \cdot (7 \cdot 3 \cdot b)} = \frac{3a}{16}$

20. $\frac{28m}{15n} \div \frac{7m^2}{3n^3} = \frac{28m \cdot 3n^3}{15n \cdot 7m^2} = \frac{4n^2}{5m}$

21. $\frac{24abc}{7xyz^2} \cdot \frac{14x^2yz}{9a^2} = \frac{3 \cdot 8 \cdot a \cdot b \cdot c \cdot 2 \cdot 7 \cdot x^2 \cdot y \cdot z}{7 \cdot x \cdot y \cdot z^2 \cdot 3 \cdot 3 \cdot a^2}$
$= \frac{8 \cdot b \cdot c \cdot 2 \cdot x(3 \cdot 7 \cdot a \cdot x \cdot y \cdot z)}{z \cdot 3 \cdot a(3 \cdot 7 \cdot a \cdot x \cdot y \cdot z)} = \frac{8 \cdot b \cdot c \cdot 2 \cdot x}{z \cdot 3 \cdot a} = \frac{16bcx}{3az}$

25. $\frac{x+y}{3} \cdot \frac{12}{(x+y)^2} = \frac{2 \cdot 2 \cdot 3(x+y)}{3(x+y)^2} = \frac{4}{x+y}$

30. $\frac{8y+16}{3-y} \cdot \frac{4y-12}{3y+6} = \frac{8(y+2) \cdot 4(y-3)}{-(y-3) \cdot 3(y+2)} = \frac{8 \cdot 4}{-3} = -\frac{32}{3}$

37. $\frac{r^2-16}{r+1} \div \frac{r+4}{r^2-1} = \frac{(r^2-16)(r^2-1)}{(r+1)(r+4)}$
$= \frac{(r-4)(r+4)(r+1)(r-1)}{(r+1)(r+4)} = \frac{(r-4)(r-1)}{1}$
$= r^2 - 5r + 4$

39. $\frac{9-x^2}{x+y} \cdot \frac{4x+4y}{x-3} = \frac{(3-x)(3+x) \cdot 4(x+y)}{(x+y)(x-3)}$
$= \frac{-1(x-3)(x+3) \cdot 4(x+y)}{(x+y)(x-3)} = \frac{-1(x+3) \cdot 4}{1}$
$= -4(x + 3) = -4x - 12$

41. $\frac{a^2-5a+6}{a^2-9a+20} \cdot \frac{a^2-5a+4}{a^2-3a+2}$
$= \frac{(a-3)(a-2) \cdot (a-4)(a-1)}{(a-4)(a-5) \cdot (a-2)(a-1)} = \frac{a-3}{a-5}$

47. $\frac{6r^2-r-7}{12r^2+16r-35} \div \frac{r^2-r-2}{2r^2+r-10}$
$= \frac{(6r^2-r-7)(2r^2+r-10)}{(12r^2+16r-35)(r^2-r-2)}$
$= \frac{(6r-7)(r+1)(2r+5)(r-2)}{(6r-7)(2r+5)(r-2)(r+1)} = 1$

49. $(3x^2 - 2x - 8) \div \frac{x^2-4}{x+2} = \frac{(3x+4)(x-2)}{1}$
$\cdot \frac{x+2}{(x+2)(x-2)} = 3x + 4$

53. $\frac{10}{a^3-27} \cdot \frac{a^2+3a-18}{15} = \frac{2 \cdot 5(a+6)(a-3)}{3 \cdot 5(a-3)(a^2+3a+9)}$
$= \frac{2(a+6)}{3(a^2+3a+9)} = \frac{2a+12}{3a^2+9a+27}$

Review exercises

1. $\frac{19}{12}$ **2.** $\frac{11}{24}$ **3.** $2(x + 5)(x - 5)$ **4.** $(x + 11)(x - 2)$
5. $(x + 4)^2$ **6.** $x = \frac{24}{5}$ **7.** $y = 15$ **8.** 7.89×10^{-5}

Exercise 6–2

Answers to odd-numbered problems

1. $\frac{8}{x}$ **3.** $\frac{7}{p}$ **5.** $\frac{14x}{x+2}$ **7.** $-\frac{2}{x}$ **9.** $\frac{x+2}{x^2-1}$

11. $-\frac{1}{7}$ **13.** $\frac{9}{z}$ **15.** $\frac{-7}{x-2}$ **17.** $\frac{9y}{y-6}$

19. $\frac{-x+4}{x-5}$ **21.** $\frac{3y+2}{2y-3}$ **23.** $\frac{-x+4}{2x-5}$ **25.** $18x$

27. $48x^2$ **29.** $140y^3$ **31.** $64a^4$ **33.** $180a^3$ **35.** $3(x-4)$

37. $18y^3(y-4)$ **39.** $a(a+1)(a-1)$ **41.** $8(a+2)(a+1)$

43. $(a+2)(a-2)(a-3)$ **45.** $(a+3)(a-3)(a-2)$

47. $2(x-7)(x+7)$

Solutions to trial exercise problems

5. $\frac{5x}{x+2}+\frac{9x}{x+2}=\frac{5x+9x}{x+2}=\frac{14x}{x+2}$ **8.** $\frac{3y-2}{y^2}-\frac{4y-1}{y^2}$

$=\frac{(3y-2)-(4y-1)}{y^2}=\frac{3y-2-4y+1}{y^2}=\frac{-y-1}{y^2}$

16. $\frac{1}{x-7}-\frac{5}{7-x}=\frac{1}{x-7}+\frac{5}{x-7}=\frac{6}{x-7}$

21. $\frac{2y-5}{2y-3}-\frac{y+7}{3-2y}=\frac{2y-5}{2y-3}+\frac{y+7}{2y-3}=\frac{3y+2}{2y-3}$

32.
$$\begin{aligned}4x^2&=2^2\cdot x^2\\3x&=3\cdot x\\8x^3&=2^3\cdot x^3\\\text{LCD}&=2^3\cdot 3\cdot x^3=24x^3\end{aligned}$$

44.
$$\begin{aligned}y^2-y-12&=(y-4)(y+3)\\y^2+6y+9&=(y+3)^2\\\text{LCD}&=(y+3)^2(y-4)\end{aligned}$$

47.
$$\begin{aligned}x^2-49&=(x+7)(x-7)\\7-x&=-1(x-7)\\2x+14&=2(x+7)\\\text{LCD}&=2(x+7)(x-7)\end{aligned}$$

Review exercises

1. commutative property of addition **2.** $5(y+2)(y-2)$

3. $(x+10)^2$ **4.** $(3y-4)(y+1)$ **5.** $\left\{\frac{8}{19}\right\}$ **6.** $\{-3,5\}$

Exercise 6–3

Answers to odd-numbered problems

1. $\frac{2x+9}{12}$ **3.** $\frac{8x+1}{48}$ **5.** $\frac{11}{3}$ **7.** $\frac{15y-12}{(y+4)(y-5)}$ **9.** $\frac{10}{y-2}$

11. $\frac{7x+62}{4(x+2)(x-2)}$ **13.** $\frac{9x+40}{x+8}$ **15.** $\frac{x(x+4)}{(x+1)(x-1)}$

17. $\frac{9x+22}{(x+3)(x-3)(x+2)}$ **19.** $\frac{7y^2-13y}{(y-3)^2(y+1)}$

21. $\frac{2y^2-9y+1}{(y-5)(y-3)(y+2)}$ **23.** $\frac{2y-15}{18}$ **25.** $\frac{25}{42y}$

27. $\frac{19a+39}{60}$ **29.** $\frac{4x+25}{18x}$ **31.** $\frac{-5x-17}{(2x-3)(x-5)}$

33. $\frac{17}{7(y+2)}$ **35.** $\frac{9x+66}{x+8}$ **37.** $\frac{-25x-9}{3x+1}$

39. $\frac{-6a+3}{(a-2)(a+2)(a-3)}$ **41.** $\frac{12y-12}{(y-6)(y+4)(y-3)}$

43. $\frac{-3a^2+11a-3}{(a-3)(a-2)}$ **45.** $\frac{13}{12z}$ **47.** $\frac{52a}{45}$ **49.** $\frac{39b-5}{36}$

51. $\frac{-z^3+9z^2-13}{(z-2)(z+2)(z-1)(z+1)}$ **53.** $\frac{bc+ac+ab}{abc}$ **55.** $\frac{3}{m}$

57. $\frac{h}{36}$ **59.** $\frac{48}{m}$ **61.** $w=\frac{A}{23}$ **63.** $b=\frac{2A}{9}$ **65.** $t=\frac{d}{55}$

67. $\frac{b}{a}$

Solutions to trial exercise problems

6. $\frac{4}{x-1}+\frac{5}{x+3}=\frac{4(x+3)+5(x-1)}{(x-1)(x+3)}$

$=\frac{4x+12+5x-5}{(x-1)(x+3)}=\frac{9x+7}{(x-1)(x+3)}$

11. $\frac{12}{x^2-4}+\frac{7}{4x-8}=\frac{12}{(x+2)(x-2)}+\frac{7}{4(x-2)}$

$=\frac{12\cdot 4}{4(x+2)(x-2)}+\frac{7(x+2)}{4(x+2)(x-2)}=\frac{48+(7x+14)}{4(x+2)(x-2)}$

$=\frac{7x+62}{4(x+2)(x-2)}$ **13.** $5+\frac{4x}{x+8}=\frac{5(x+8)}{x+8}+\frac{4x}{x+8}$

$=\frac{5x+40+4x}{x+8}=\frac{9x+40}{x+8}$ **25.** $\frac{9}{14y}-\frac{1}{21y}=\frac{9\cdot 3}{2\cdot 3\cdot 7\cdot y}$

$-\frac{1\cdot 2}{2\cdot 3\cdot 7\cdot y}=\frac{27}{42y}-\frac{2}{42y}=\frac{27-2}{42y}=\frac{25}{42y}$

27. $\frac{5a+3}{12}-\frac{a-4}{10}=\frac{5(5a+3)}{60}-\frac{6(a-4)}{60}$

$=\frac{(25a+15)-(6a-24)}{60}=\frac{25a+15-6a+24}{60}=\frac{19a+39}{60}$

34. $\frac{14}{5x-15}-\frac{8}{2x-6}=\frac{14}{5(x-3)}-\frac{8}{2(x-3)}=\frac{14\cdot 2}{10(x-3)}$

$-\frac{8\cdot 5}{10(x-3)}=\frac{28-40}{10(x-3)}=\frac{-12}{10(x-3)}=\frac{-6}{5(x-3)}$

42. $\frac{2p}{p^2-9p+20}-\frac{5p-2}{p-5}=\frac{2p}{(p-5)(p-4)}-\frac{5p-2}{p-5}$

$=\frac{2p}{(p-5)(p-4)}-\frac{(5p-2)(p-4)}{(p-5)(p-4)}=\frac{2p-(5p^2-22p+8)}{(p-5)(p-4)}$

$=\frac{2p-5p^2+22p-8}{(p-5)(p-4)}=\frac{-5p^2+24p-8}{(p-5)(p-4)}$

52. $\frac{1}{f}=(u-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=(u-1)\left(\frac{R_2}{R_1R_2}+\frac{R_1}{R_1R_2}\right)$

$=(u-1)\left(\frac{R_2+R_1}{R_1R_2}\right)$ **59.** Since n is one of the numbers, let x be the other number. Then $x\cdot n=48$ and x (the other number) $=\frac{48}{n}$.

62. Using $A=\frac{1}{2}bh$, then $2A=bh$ (multiply each member by 2).

Then $h=\frac{2A}{b}=\frac{2(21)}{b}=\frac{42}{b}$.

Review exercises

1. $(x-7)^2$ **2.** $(2x-1)(x-5)$ **3.** $4(x+2)(x-2)$

4. x^2-81 **5.** $16x^2+24x+9$ **6.** $2x^2-15x-8$

7. 48 **8.** $12x^2$ **9.** $(x-3)^2(x+3)$

10. $\frac{13}{2x}$ **11.** $\frac{2x+1}{(x-2)(x-1)}$

Exercise 6–4

Answers to odd-numbered problems

1. $\frac{5}{6}$ **3.** $\frac{3}{2}$ **5.** $\frac{8}{9}$ **7.** $\frac{5}{2}$ **9.** $\frac{23}{30}$ **11.** $\frac{11}{2}$

13. $\frac{4x+1}{4x-3}$ **15.** $\frac{3a+1}{2-4a}$ **17.** $\frac{4a^2+3}{5a^2-3a}$ **19.** $\frac{ab-3}{ab+4}$

21. $\frac{y+x}{y-x}$ **23.** xy **25.** $\frac{4b-5a^2}{a^3b-a^2b^2}$ **27.** $-\frac{y}{x}$

29. $\frac{2(x^2+y^2)}{(x^2-y^2)^2}$ **31.** $\frac{2b+3}{a^3b^4}$ **33.** $\frac{3a-1}{17-a}$ **35.** $\frac{-5}{x+31}$

37. $-\frac{3a^2+5a-8}{a+29}$ **39.** $\frac{(y-3)(y+6)}{(y+1)(y-4)}$ **41.** $\frac{T_1}{T_2-T_1}$

43. $\frac{L_1L_2L_3}{L_2L_3+L_1L_3+L_1L_2}$

Solutions to trial exercise problems

7. $$\frac{7}{2+\frac{4}{5}} = \frac{7\cdot 5}{\left(2+\frac{4}{5}\right)\cdot 5} = \frac{7\cdot 5}{2\cdot 5+\frac{4}{5}\cdot 5} = \frac{35}{10+4} = \frac{35}{14} = \frac{5}{2}$$

12. $$\frac{\frac{6}{7}-\frac{5}{14}}{\frac{3}{14}-\frac{5}{7}} = \frac{\left(\frac{6}{7}-\frac{5}{14}\right)\cdot 14}{\left(\frac{3}{14}-\frac{5}{7}\right)\cdot 14} = \frac{\frac{6}{7}\cdot 14-\frac{5}{14}\cdot 14}{\frac{3}{14}\cdot 14-\frac{5}{7}\cdot 14} = \frac{12-5}{3-10}$$
$$= \frac{7}{-7} = -1$$

15. $$\frac{\frac{1}{a}+3}{\frac{2}{a}-4} = \frac{\left(\frac{1}{a}+3\right)\cdot a}{\left(\frac{2}{a}-4\right)\cdot a} = \frac{\frac{1}{a}\cdot a+3\cdot a}{\frac{2}{a}\cdot a-4\cdot a} = \frac{1+3a}{2-4a} = \frac{3a+1}{2-4a}$$

22. $$\frac{\frac{3}{x^2}-\frac{4}{y}}{\frac{5}{x}+\frac{2}{y^2}} = \frac{\left(\frac{3}{x^2}-\frac{4}{y}\right)\cdot x^2y^2}{\left(\frac{5}{x}+\frac{2}{y^2}\right)\cdot x^2y^2} = \frac{\frac{3}{x^2}\cdot x^2y^2-\frac{4}{y}\cdot x^2y^2}{\frac{5}{x}\cdot x^2y^2+\frac{2}{y^2}\cdot x^2y^2}$$
$$= \frac{3y^2-4x^2y}{5xy^2+2x^2}$$

27. $$\frac{\frac{1}{x+y}-\frac{1}{x-y}}{\frac{1}{x+y}+\frac{1}{x-y}} = \frac{\left(\frac{1}{x+y}-\frac{1}{x-y}\right)\cdot(x+y)(x-y)}{\left(\frac{1}{x+y}+\frac{1}{x-y}\right)\cdot(x+y)(x-y)}$$
$$= \frac{\frac{1}{x+y}\cdot(x+y)(x-y)-\frac{1}{x-y}\cdot(x+y)(x-y)}{\frac{1}{x+y}\cdot(x+y)(x-y)+\frac{1}{x-y}\cdot(x+y)(x-y)}$$
$$= \frac{(x-y)-(x+y)}{(x-y)+(x+y)} = \frac{x-y-x-y}{x-y+x+y} = \frac{-2y}{2x} = \frac{-y}{x}$$

36. $$\frac{\frac{7}{b-7}+\frac{8}{b-5}}{\frac{6}{b^2-12b+35}} = \frac{\left(\frac{7}{b-7}+\frac{8}{b-5}\right)\cdot(b-5)(b-7)}{\frac{6}{(b-5)(b-7)}\cdot(b-5)(b-7)}$$

The LCD is $(b-5)(b-7)$.

$$= \frac{\frac{7}{b-7}\cdot(b-5)(b-7)+\frac{8}{b-5}\cdot(b-5)(b-7)}{6}$$
$$= \frac{7(b-5)+8(b-7)}{6} = \frac{7b-35+8b-56}{6} = \frac{15b-91}{6}$$

Review exercises

1. $\frac{6}{7}$ **2.** $\frac{x-7}{x+2}$ **3.** $2x^3+3x^2+x-21$ **4.** $\{3\}$ **5.** $\{-2\}$
6. $\{-3,1\}$ **7.** Domain is all real numbers except -7. **8.** Domain is all real numbers except -2 and 2. **9. a.** undefined **b.** 0

Exercise 6–5

Answers to odd-numbered problems

1. $\left\{\frac{8}{3}\right\}$ **3.** $\left\{\frac{14}{3}\right\}$ **5.** $\{-22\}$ **7.** $\left\{\frac{10}{9}\right\}$ **9.** $\left\{\frac{9}{2}\right\}$

11. $\left\{-\frac{5}{8}\right\}$ $x \neq 0$ **13.** $\left\{\frac{19}{24}\right\}$ $R \neq 0$ **15.** $\left\{-\frac{5}{27}\right\}$, $y \neq 0$

17. $\{1\}$ $b \neq 0$ **19.** $\left\{\frac{4}{37}\right\}$ $p \neq 0$ **21.** $\left\{-\frac{13}{2}\right\}$ $a \neq 0$

23. $\{36\}$ $x \neq 4,-4$ **25.** $\left\{\frac{47}{9}\right\}$ $z \neq \frac{-1}{3}$ **27.** $\emptyset$; $y \neq 2$

29. $\left\{\frac{11}{5}\right\}$ $b \neq 2,-2$ **31.** $\{40\}$ $a \neq 5,-5$

33. $\left\{-\frac{19}{3}\right\}$ $x \neq 2,-3,-5$ **35.** $\left\{\frac{8}{3}\right\}$ $a \neq 3,-4$ **37.** $\left\{-\frac{2}{3}\right\}$

39. $\left\{\frac{3}{2},-3\right\}$ **41.** $\left\{\frac{2}{3},-4\right\}$ **43.** $\left\{\frac{2}{3},-1\right\}$

45. $x = \frac{2y}{3y-1}$ **47.** $c_1 = \frac{cc_2}{c_2-c}$ **49.** $a = \frac{8b}{b-8}$

51. $a = \frac{3b-5}{4}$ **53.** $x = \frac{-2(y+6)}{5}$ **55.** $D = \frac{P}{pL}$

57. $N = \frac{2P}{D-P}$ **59.** $f = \frac{Fa}{A}$ **61.** $T_2 = \frac{T_1P_2V_2}{P_1V_1}$

63. $L_1 = \frac{L_2}{\alpha(t_2-t_1)+1}$ **65.** $R_1 = \frac{RR_2}{R_2-R}$

Solutions to trial exercise problems

2. $\frac{4x}{5}-\frac{2}{3} = 4$ Multiply by 15 to get

The LCD is 15.
$$\begin{aligned} \frac{4x}{5}\cdot 15-\frac{2}{3}\cdot 15 &= 4\cdot 15 \\ 4x\cdot 3-2\cdot 5 &= 60 \\ 12x-10 &= 60 \\ 12x &= 70 \\ x &= \frac{70}{12} = \frac{35}{6} \end{aligned}$$

The solution set is $\left\{\frac{35}{6}\right\}$.

6. $\frac{3R}{4}-5 = \frac{5}{6}$ Multiply by 12 to get

The LCD is 12.
$$\begin{aligned} \frac{3R}{4}\cdot 12-5\cdot 12 &= \frac{5}{6}\cdot 12 \\ 3R\cdot 3-60 &= 5\cdot 2 \\ 9R-60 &= 10 \\ 9R &= 70 \\ R &= \frac{70}{9} \end{aligned}$$

The solution set is $\left\{\frac{70}{9}\right\}$.

15. $\dfrac{4}{6y} + 5 = \dfrac{1}{9y} + 2$ Multiply by $18y$ to get

The LCD is $18y$.
$$\begin{aligned}\frac{4}{6y}\cdot 18y + 5\cdot 18y &= \frac{1}{9y}\cdot 18y + 2\cdot 18y\\ 4\cdot 3 + 90y &= 1\cdot 2 + 36y\\ 12 + 90y &= 2 + 36y\\ 90y &= -10 + 36y\\ 54y &= -10\\ y &= \frac{-10}{54} = \frac{-5}{27}\end{aligned}$$

The solution set is $\left\{-\dfrac{5}{27}\right\}(y \neq 0)$.

22. $\dfrac{R+2}{10R} + \dfrac{4R-1}{4R} = 2$ Multiply by $20R$ to get

The LCD is $20R$.
$$\begin{aligned}\frac{R+2}{10R}\cdot 20R + \frac{4R-1}{4R}\cdot 20R &= 2\cdot 20R\\ (R+2)\cdot 2 + (4R-1)\cdot 5 &= 40R\\ 2R + 4 + 20R - 5 &= 40R\\ 22R - 1 &= 40R\\ -1 &= 18R\\ R &= \frac{-1}{18}\end{aligned}$$

The solution set is $\left\{-\dfrac{1}{18}\right\}(R \neq 0)$.

34. $\dfrac{8}{a^2 - 6a + 8} = \dfrac{1}{a^2 - 16}$ Multiply both members by $(a-2)(a+4)(a-4)$.
$$\begin{aligned}\frac{8}{(a-4)(a-2)}\cdot(a-2)(a+4)(a-4) &= \frac{1}{(a+4)(a-4)}\cdot(a-2)(a+4)(a-4)\\ 8(a+4) &= 1(a-2)\\ 8a + 32 &= a - 2\\ 8a &= a - 34\\ 7a &= -34\\ a &= \frac{-34}{7}\end{aligned}$$

The solution set is $\left\{-\dfrac{34}{7}\right\}(a \neq 4,2,-4)$.

37. $3x^2 + 4x + \dfrac{4}{3} = 0$ Multiply by 3 to obtain

$9x^2 + 12x + 4 = 0$ Factor the left member.
$$\begin{aligned}(3x+2)^2 &= 0\\ 3x + 2 &= 0\\ 3x &= -2\\ x &= -\frac{2}{3}\end{aligned}$$
So

The solution set is $\left\{-\dfrac{2}{3}\right\}$.

42. $x^2 - \dfrac{5}{6}x = \dfrac{2}{3}$ Multiply by the LCD, 6, to obtain
$$\begin{aligned}6x^2 - 5x &= 4\\ 6x^2 - 5x - 4 &= 0\\ (2x+1)(3x-4) &= 0\\ 2x + 1 = 0 &\text{ or } 3x - 4 = 0\\ x = -\frac{1}{2} &\text{ or } x = \frac{4}{3}\end{aligned}$$

The solution set is $\left\{-\dfrac{1}{2}, \dfrac{4}{3}\right\}$.

47. $\dfrac{1}{c} = \dfrac{1}{c_1} + \dfrac{1}{c_2}$ Multiply by cc_1c_2.

$$\frac{1}{c} \cdot cc_1c_2 = \frac{1}{c_1} \cdot cc_1c_2 + \frac{1}{c_2} \cdot cc_1c_2$$

$c_1c_2 = cc_2 + cc_1$ Add $-cc_1$ to both members.

$c_1c_2 - cc_1 = cc_2$ Factor c_1 in the left member.

$(c_2 - c)c_1 = cc_2$ Divide both members by $c_2 - c$.

$$c_1 = \frac{cc_2}{c_2 - c}$$

52. $\dfrac{5}{m} + 4 = \dfrac{6a}{2m} + 3b$ Multiply by the LCD $2m$.

$$\frac{5}{m} \cdot 2m + 4 \cdot 2m = \frac{6a}{2m} \cdot 2m + 3b \cdot 2m$$

$10 + 8m = 6a + 6bm$ Add -10 to both members.

$8m = 6a - 10 + 6bm$ Add $-6bm$ to both members.

$8m - 6bm = 6a - 10$ Factor m in the left member.

$(8 - 6b)m = 6a - 10$ Divide both members by $8 - 6b$.

$$m = \frac{6a - 10}{8 - 6b} = \frac{2(3a - 5)}{2(4 - 3b)} = \frac{3a - 5}{4 - 3b}$$

58. $\dfrac{T_A}{T_B} = \dfrac{R_B}{R_A}$ Multiply by the LCD T_BR_A.

$$\frac{T_A}{T_B} \cdot T_BR_A = \frac{R_B}{R_A} \cdot T_BR_A$$

$$T_AR_A = R_BT_B$$

$$T_B = \frac{T_AR_A}{R_B}$$

62. Given $k = \dfrac{L_t - L_0}{L_0t}$. Multiply both members by L_0t to get $kL_0t = L_t - L_0$.

a. To find L_t, add L_0 to both members to get $L_t = kL_0t + L_0$.

b. To find L_0, use $L_t = kL_0t + L_0$, factor L_0 in the right member to get $L_t = (kt + 1)L_0$. Divide both members by $kt + 1$.

Then $\dfrac{L_t}{kt + 1} = L_0$ or $L_0 = \dfrac{L_t}{kt + 1}$.

Review exercises

1. 14 **2.** $\dfrac{17}{3}$ **3.** 4

4. $y = -5x + 4$ **5.** $y = \dfrac{-2x + 6}{3}$

6. $y = \dfrac{x - 8}{4}$ **7.** $\dfrac{3y}{x^2}$ **8.** $\dfrac{y^4}{x^5}$

Exercise 6–6

Answers to odd-numbered problems

1. $29\dfrac{1}{6}$ minutes **3.** $2\dfrac{14}{29}$ hours **5.** 12 hours **7.** $3\dfrac{1}{3}$ hours

9. 10 hours **11.** 110 miles at 55 mph, 210 miles at 60 mph

13. car *A*, 40 mph; car *B*, 50 mph **15.** $\dfrac{6}{7}$ mph **17.** 220 miles

19. $\dfrac{5}{9}$ **21.** 3 and 12 **23.** 6 **25.** 4 **27.** adult, 15 pills; child, 9 pills **29.** \$38.21 **31.** $\dfrac{60}{17}$ ohms or $3\dfrac{9}{17}$ ohms **33.** 20 ohms

35. 60 ohms

Solutions to trial exercise problems

1. Let x = the number of minutes required for both boys to mow the lawn. Then,

$$\frac{1}{50} + \frac{1}{70} = \frac{1}{x}$$

Multiply by the LCM of 50, 70, and x, that is, $350x$.

$$350x \cdot \frac{1}{50} + 350x \cdot \frac{1}{70} = 350x \cdot \frac{1}{x}$$
$$7x + 5x = 350$$
$$12x = 350$$
$$x = \frac{350}{12} = 29\frac{1}{6}$$

Together Jim and Kenny could mow the lawn in $29\frac{1}{6}$ minutes.

5. Let x = the time required for Dick to paint the house alone.

Then $\frac{1}{6} + \frac{1}{x} = \frac{1}{4}$.

Multiply both members by the LCM of 4, 6, and x, that is, $12x$.

$$12x \cdot \frac{1}{6} + 12x \cdot \frac{1}{x} = 12x \cdot \frac{1}{4}$$
$$2x + 12 = 3x$$
$$12 = x$$

Therefore Dick would take 12 hours to paint the house alone.

13. Let x = the average speed of car A.

Then $x + 10$ = the average speed of car B.

Using $t = \frac{d}{r}$, let t_A = time of car A, t_B = time of car B,

then $t_A = t_B$.

Now $t_A = \frac{120}{x}$ and $t_B = \frac{150}{x + 10}$, so

$$\frac{120}{x} = \frac{150}{x + 10}$$

Multiply both members by the LCM of x and $x + 10$, that is, $x(x + 10)$.

$$x(x + 10) \cdot \frac{120}{x} = x(x + 10) \cdot \frac{150}{x + 10}$$
$$(x + 10)120 = 150x$$
$$120x + 1{,}200 = 150x$$
$$1{,}200 = 30x$$
$$40 = x$$

So $x + 10 = 50$. Therefore car B averaged 50 mph and car A averaged 40 mph.

20. Let x = the numerator of the fraction. Then $x + 7$ = the denominator of the fraction.

Then $\frac{x + 3}{(x + 7) - 1} = \frac{4}{5}$. Simplifying, we have the equation

$$\frac{x + 3}{x + 6} = \frac{4}{5}.$$

Multiply both members by the LCM of 5 and $x + 6$, that is, $5(x + 6)$.

$$5(x + 6) \cdot \frac{x + 3}{x + 6} = 5(x + 6) \cdot \frac{4}{5}$$
$$5(x + 3) = 4(x + 6)$$
$$5x + 15 = 4x + 24$$
$$x + 15 = 24$$
$$x = 9$$
$$x + 7 = 16$$

The original fraction is then $\frac{9}{16}$.

21. Let x = the lesser of the two numbers then

$4x$ = the greater of the two numbers.

Their reciprocals are then $\frac{1}{x}$ and $\frac{1}{4x}$ and we get the equation

$$\frac{1}{x} + \frac{1}{4x} = \frac{5}{12}.$$

Multiply both members by the LCM of x, $4x$, and 12, that is, $12x$.

$$12x \cdot \frac{1}{x} + 12x \cdot \frac{1}{4x} = 12x \cdot \frac{5}{12}$$
$$12 + 3 = 5x$$
$$15 = 5x$$

then
$$x = 3$$
$$4x = 12$$

Therefore the two numbers are 3 and 12.

$\left(\textit{Note: } \text{To check, show } \frac{1}{3} + \frac{1}{12} = \frac{5}{12}\right.$

$$\frac{4}{12} + \frac{1}{12} = \frac{5}{12}$$
$$\left.\frac{5}{12} = \frac{5}{12}\right)$$

28. Let x = hourly wage of a journeyman electrician, then

$\frac{3}{8}x$ = the hourly wage of an apprentice electrician.

$$x + \frac{3}{8}x = 29.70$$
$$8 \cdot x + 8 \cdot \frac{3}{8}x = 8 \cdot 29.70$$
$$8x + 3x = 237.60$$
$$11x = 237.60$$
$$x = \$21.60$$
$$\frac{3}{8}x = \$8.10$$

The journeyman earns \$21.60 per hour and the apprentice earns \$8.10 per hour.

30. Let R = the total resistance of the circuit. Then

$$\frac{1}{6} + \frac{1}{8} = \frac{1}{R}.$$

Multiply by the LCM of 6, 8, and R, that is, $24R$.

$$24R \cdot \frac{1}{6} + 24R \cdot \frac{1}{8} = 24R \cdot \frac{1}{R}$$
$$4R + 3R = 24$$
$$7R = 24$$
$$R = \frac{24}{7}$$

Therefore the total resistance in the parallel circuit is $\frac{24}{7}$ or $3\frac{3}{7}$ ohms.

33. Let R_1 = the resistance in the unknown branch. Then

$$\frac{1}{R_1} + \frac{1}{30} = \frac{1}{12}.$$

Multiply by the LCM of R_1, 12, and 30, that is, $60R_1$.

$$60R_1 \cdot \frac{1}{R_1} + 60R_1 \cdot \frac{1}{30} = 60R_1 \cdot \frac{1}{12}$$
$$60 + 2R_1 = 5R_1$$
$$60 = 3R_1$$
$$20 = R_1$$

Therefore the other branch has resistance of 20 ohms.

Review exercises

1. $\{2\}$ **2.** $y = \frac{3x+6}{2}$ **3.** $8(y+2)(y-2)$

4. $(x+10)^2$ **5.** $(3y+2)(y-2)$ **6.** $\frac{5x^2+7x}{(x-1)(x+3)}$

7. $\frac{-2y^2-23y}{(2y+1)(y-5)}$

Chapter 6 review

1. $\frac{9}{2}$ **2.** $\frac{9a}{b}$ **3.** $\frac{2x-y}{2x+y}$ **4.** $x-2$ **5.** $\frac{x(m-n)}{2}$

6. $\frac{1}{y^2-2y+1}$ **7.** 2 **8.** $-\frac{1}{3x+15}$ **9.** $\frac{x-1}{x+1}$

10. $\frac{2a}{3}$ **11.** $\frac{8b}{21}$ **12.** $\frac{9}{2ab}$ **13.** $\frac{14x-7}{9x+12}$

14. $\frac{1}{x^2+4x-12}$ **15.** $\frac{10}{3}$ **16.** $\frac{1}{x-8}$ **17.** $\frac{3-b}{b^2+2b-3}$

18. $\frac{3a+3}{4a-4}$ **19.** $36x^2$ **20.** $42a^2b^2$ **21.** $2(x+3)(x-3)$

22. $(y+5)(y-5)(y+3)$ **23.** $4(z+2)(z-2)(z+1)$

24. $x(x+1)^2(3x-5)$ **25.** $\frac{3}{x}$ **26.** $\frac{4y}{y-2}$ **27.** $\frac{4x+4}{3x-1}$

28. $\frac{-x+13}{2x-5}$ **29.** $\frac{68x}{105}$ **30.** $\frac{23}{48a}$

31. $\frac{47x-6}{(3x+1)(4x-3)}$ **32.** $\frac{4x^2-23x}{(x+4)(x-4)}$

33. $\frac{80a+48b-15ab}{20a^2b^2}$ **34.** $\frac{4a^2+4a+6}{a+1}$ **35.** $\frac{-10x^2-3}{x^2+1}$

36. $\frac{16x^2+5x+15}{24x^2}$ **37.** $\frac{13y^2-89y}{(y-9)(y+2)(y-2)}$

38. $\frac{2-2ax-36a}{(x+3)(x-3)}$ **39.** $\frac{8x^2-40x-180}{x(x-5)(x+4)}$

40. $\frac{10x-5y}{(x+y)^2(x-2y)}$ **41.** $\frac{16}{63}$ **42.** $\frac{1}{7}$ **43.** 1 **44.** $\frac{y-x}{y+x}$

45. $\frac{4+3x}{2-5x}$ **46.** $\frac{a^2-b^2}{a^2+b^2}$ **47.** $y-x$ **48.** $\frac{xy}{x+y}$

49. $\frac{a^3-2a^2b^2-3ab^2}{ab-3b^2}$ **50.** $\{-96\}$ **51.** $\left\{\frac{41}{48}\right\}$ **52.** $\left\{\frac{13}{24}\right\}$

53. $\left\{\frac{6}{5}\right\}$ **54.** $\left\{-\frac{39}{14}\right\}$ **55.** $\left\{\frac{7}{52}\right\}$ **56.** $\{0,12\}$

57. $\left\{-\frac{3}{2},\frac{3}{2}\right\}$ **58.** $\{-5,3\}$ **59.** $\left\{6,\frac{3}{2}\right\}$ **60.** $x = \frac{a+b}{3}$

61. $x = \frac{y+3}{1-y}$ **62.** $y = \frac{4a-3b}{a}$ **63.** $y = 4b+5c$

64. $l = \frac{Aey}{F}$ **65.** $L = \frac{Wp-2E}{EF}$ **66.** $R = \frac{Mr}{M-2}$

67. $m = \frac{Fr}{v^2+gr}$ **68.** $5\frac{1}{3}$ hr, 16 hr **69.** $1\frac{5}{7}$ mph **70.** $\frac{2}{3}$ or 1

Chapter 6 cumulative test

1. $-\frac{71}{12}$ **2.** $\frac{3}{4}$ **3.** $\frac{43}{30}$ **4.** $\frac{8}{5}$ **5.** x^{11} **6.** $\frac{1}{y^9}$ **7.** $\frac{25x^4}{y^6}$

8. $10x^3-4x^2+11x-9$ **9.** $\left\{-\frac{26}{7}\right\}$ **10.** $\left\{\frac{81}{4}\right\}$

11. $\left\{-\frac{3}{2},2\right\}$ **12.** $x \geq -5$ **13.** $2(x+3)(x-3)$

14. $3x^2(2x^3-12x+3)$ **15.** $(2x+5)(2x+3)$

16. $3(1+2x^3)(1-2x^3)$ **17.** $49x^2-84x+36$

18. $25-\frac{1}{4}x^2$ **19.** $6x^3-19x^2+37x-33$

20. $15-22x^2+8x^4$ **21.** $x = \frac{120}{7}$

22. 2,400 foot-pounds/min **23.** $\frac{4a^2cf\ell}{5b^2}$ **24.** $\frac{y^2+10y+21}{y^2-8y+12}$

25. y^2-5y+6 **26.** $\frac{(x-y)(x-1)}{y}$ **27.** $\frac{2y+4x}{x^2y^2}$

28. $\frac{3x+9y}{(x-y)(x+y)}$ **29.** $\frac{9}{a-2}$ **30.** $\frac{13x+56}{(x-7)(x+7)(x+2)}$

31. $\frac{-x^2-6x}{(x+3)(x-2)(x-6)}$ **32.** $\frac{3y-1}{4y+5}$ **33.** $\frac{y+x}{3x-4y}$

34. $\frac{x^2-11x+28}{x-5}$ **35.** $\{2\}$ **36.** $\left\{-\frac{1}{3},6\right\}$ **37.** $q = \frac{fp}{p-f}$

38. $2\frac{2}{5}$ days **39.** $\frac{8}{16}$

Chapter 7

Exercise 7-1

Answers to odd-numbered problems

1. (1,2) and $(-1,-4)$ are solutions. **3.** $(-1,2)$, (3,0) are solutions. **5.** (1,2) is a solution. **7.** (2,3), (0,0) are solutions. **9.** $(-4,1)$ and $(-4,-4)$ are solutions. **11.** $(-5,3)$, $(-5,8)$ are solutions. **13.** (1,5), $(-2,-4)$, (0,2) **15.** $(3,-3)$, $(-4,11)$, (0,3)

17. $(3,-5)$, $(-2,10)$, (0,4) **19.** $(-2,-1)$, (3,0), $\left(0,-\frac{3}{5}\right)$

21. $(1,-4)$, $(-1,1)$, $\left(0,-\frac{3}{2}\right)$ **23.** (1,5), $(-6,5)$, (0,5)

25. $(7,-1)$, $\left(-\frac{3}{5},-1\right)$, $(0,-1)$ **27.** (5,1), $(-1,-2)$, (3,0)

29. (3,2), $(-6,-4)$, (0,0) **31.** $(1,-2)$, (1,7), (1,0) **33. a.** (75,\$170) **b.** (300,\$620) **c.** (1,000,\$2,020) **35. a.** (2,236) **b.** (12,216) **c.** (0,240) **37. a.** (0,20) **b.** (5,35) **c.** (3,29)

39. a. (3,165) **b.** (8,440) **c.** $\left(\frac{26}{5},286\right)$

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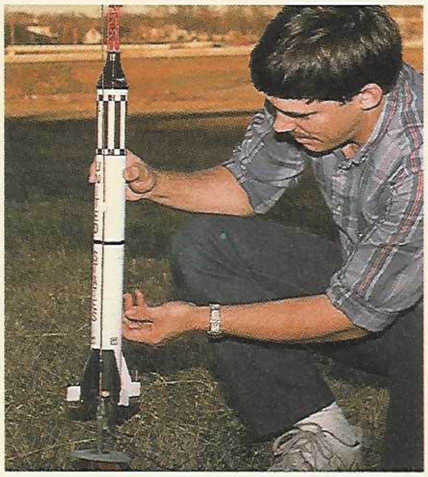

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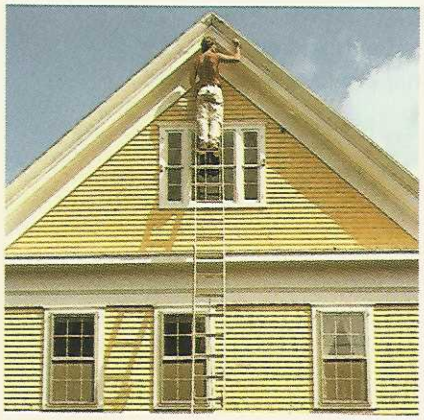

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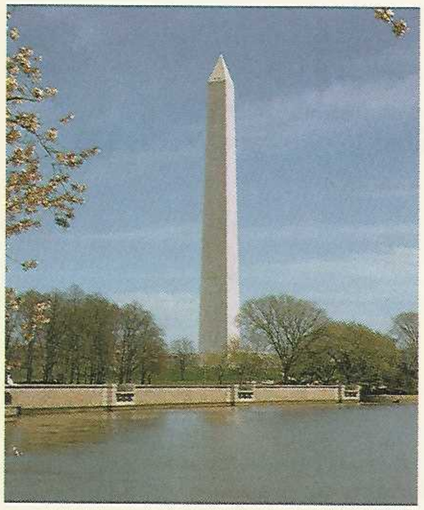

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